Engaging Elementary Students in Geometry and Measurement

BY ROBERT C. MOORE

I will never forget Jason. His fourth-grade teacher, Ms. Rowland, and I were teaching a geometry lesson on squares. We began the lesson by giving the children some color-coded plastic sticks with flexible connectors, called D-Stix. Without telling them what a square was, we asked them to make some squares of different sizes. Most of the children immediately accomplished the task, but Jason chose four sticks of different lengths and made a quadrilateral. Because it didn't look right, he tried other combinations of sticks. Then suddenly his face brightened, and he chose four sticks of equal length to make his square. I will always remember that moment, and Jason will always remember that a square has four equal sides. However, his learning depended a great deal on the way he came to know it.

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The National Council of Teachers of Mathematics,[1] the National Research Council,[2] and other organizations have called for changes in mathematics teaching and learning. These reports recommend emphasizing different concepts: less time spent simply naming geometric figures and more time learning geometric properties and relationships, less time spent memorizing equivalencies between units of measurement and more time spent on estimating and

hands-on measuring. These reports also challenge us to try new teaching methods, for an underlying premise of the recommendations "is that *what* a student learns depends to a great degree on *how* he or she has learned it."[3] Consequently, the reports set forth a vision of mathematics classrooms as places where students explore problems and situations, make conjectures, build arguments, and discuss their various solutions.

Children thus become active learners who construct mathematical knowledge through their activities, often working in small groups. Teachers become facilitators of the learning environment who guide individual, small-group, and whole-class work, orchestrate classroom discussions, and select tasks to engage students' interests and deepen their understanding of mathematics and its applications.[4]

I met Jason and Ms. Rowland when I was working on a geometry and measurement curriculum project for grades K-6. Researchers at the University of Georgia and classroom teachers at South Jackson Elementary School (Jackson County, Georgia) cooperated in producing 158 lessons that seek to engage elementary students and teachers in the kind of active learning and teaching described above.

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This article will share a few of the project's ideas. The lesson ideas described below provide some insight into the project's instructional approach and underlying beliefs about the teaching and learning of mathematics. (For the complete set of lessons, including suggested grade-level packages, write to: Chairman, Department of Mathematics Education, 105 Aderhold Hall, University of Georgia, Athens, GA 30602, U.S.A.)

Activity A: Classification of Plane Figures

Figure 1 (page 36) shows a collection of geometric shapes. Some properties of these shapes vary (e.g., number of sides and angle sizes) while others are shared (e.g., right angles or parallel sides). You may wish to make your own collection of shapes.

Put the shapes on cards, one shape per card. Organize the children into small groups, and give each group a collection of shapes to sort. Which shapes do they think go together? Why? Have them sort the shapes in different ways; for example, by sorting the cards into two or three groups. Have them talk about their groups. Older students can write descriptions of their groups.

Follow-up activity: Have the children draw three different quadrilaterals (or triangles, etc.) Ask them to tell how the shapes differ.

Discussion. This activity can reveal a great deal about your students' understanding of geometry. I taught this lesson to a classroom of third and fourth graders. Using what I learned about their thinking, I then presented a series of lessons on angles, triangles, and

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quadrilaterals.

Notice that the activity goes beyond merely naming polygons to investigating their properties, such as the number of sides and angles, lengths of sides, sizes of angles, area, and symmetry. Adaptable to a range of grade levels, this activity allows for creativity, since many correct responses are possible. The discussion and writing will develop the children's communication skills and their use of mathematical language.

The next activity is similar, but deals with space figures rather than plane figures. Because the everyday objects used in the lesson are less abstract than the plane figures, it can be used with younger children, even kindergartners.

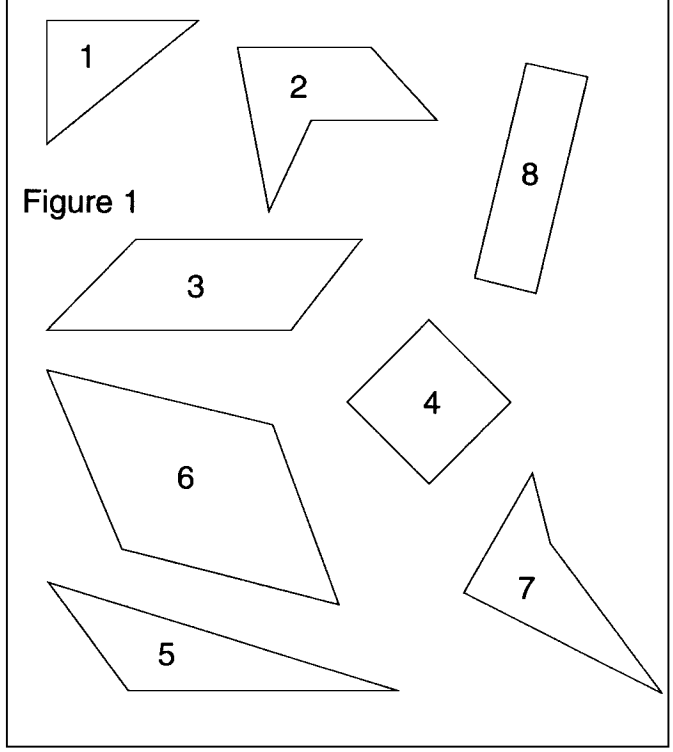


Figure 1

Activity B: Classification of Space Figures

Collect various three-dimensional shapes (cracker box, wooden block, paper cup, paper cone, can, quarter, pencil, party hat, etc.) and place them in a bag. As in the preceding activity, ask the children to sort the shapes in any way they wish. Ask them to tell how the shapes are alike and how they are different. You might blindfold the children and have them describe or classify the shapes by feeling them. Older children can make a chart of the shapes and the number of vertices, edges, and faces.

Follow-up activity: Have the children cut small paper bags along the fold lines and lay them flat. What shapes result? Challenge them to cut the bags in a variety of ways to get as many different flat shapes as they can. Have them also cut open paper cups and cones—but first ask them to draw what they think the flat shapes will look like. Soak a toilet paper core in water to soften the glue, then peel it open along the spiral seam. What flat shape results? Reversing the process, challenge the students to make three-dimensional shapes from a flat sheet of paper.

Discussion: As in the first activity, a variety of responses are possible. Concepts that may arise include vertices (or corners), edges, and faces. It's less important how the children group the objects than what properties of the shapes they perceive and why they classify the shapes as similar or different. The human side of mathematics will come into play if you let the children decide whether a face must be flat (e.g., is the curved surface of a cone a face?), and whether an edge must be straight (e.g., does a cylinder have two circular edges or no edges?). Making three-dimensional shapes from a sheet of paper can be a challenging problem-solving activity. This can be incorporated into an art lesson.

To develop your students' estimation skills, incorporate estimation in every measuring task.

The main purpose of the next activity is to help children develop concepts of length, especially "foot" and "inch" (or "meter" and "centimeter"). I have used it successfully with first graders, but other ages can benefit from it, too.

Activity C: Making an Inch or Centimeter Ruler

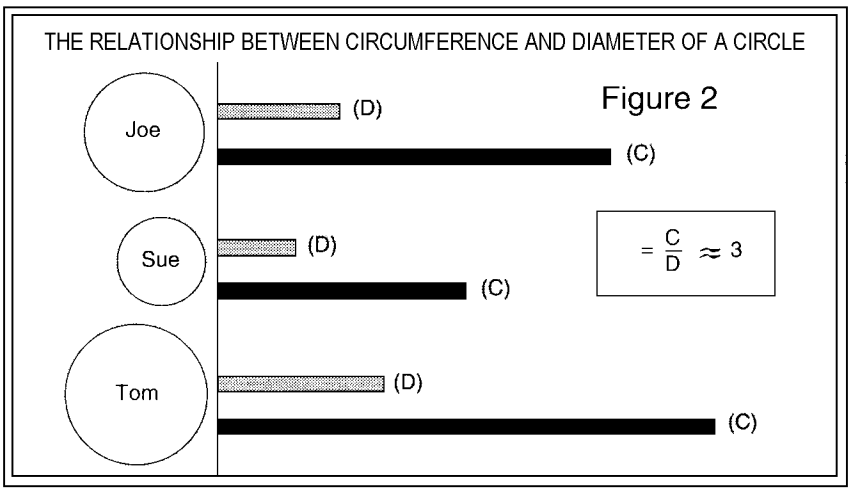
Cut strips of poster board 1 1/2 inches by 12 inches (or 2.5 cm by 10 cm). Make no marks on the strips. Give these "rulers" to the children and have them write their names on them. Ask them to find things in the room shorter or longer than the ruler or about the same length. Next, have them visually estimate the lengths of various objects, using the ruler as a unit. For example, how many rulers long is the chalkboard? If they work in pairs, they can "leap frog" their rulers. Have them record their estimates and measurements on a sheet of paper. Be sure they record their estimate of an object's length *before* they measure it.

Now give each child 12 paper inch-squares, six of one color and six of another color. (For centimeter rulers, give each child 10 rectangles 1 cm by 2 cm, five of one color and five of another color. Centimeter squares are too small for children to handle well.) Have them glue these squares along one edge of their rulers, alternating the colors. (Tell them not to write numbers on the rulers.) Have them measure a variety of objects with their rulers and record the results.

Discussion: Children learn measurement by measuring. They need to engage in many measuring activities. This lesson develops their concept of foot and inch, or decimeter and centimeter. The paper squares show that a foot is divided into 12 spaces (a decimeter into 10 spaces), and that when we measure we are counting *spaces*, not tick marks on a ruler.

The lesson also helps children learn the processes of measuring. Help them to measure accurately. If two groups get different answers for the length of the chalkboard, have them discuss why. To develop your students' estimation skills,

incorporate estimation in every measuring task. You can bring addition or multiplication into the lesson by asking questions like this: If the chalkboard is 6 feet 3 inches long, how many inches long is it? Keeping a record sheet is a step toward learning to collect and analyze data, an important skill in modern society.



The next two activities integrate measurement with geometry, the first through a study of circles, the second through the study of cylinders. Third and fourth graders enjoy the circle lesson. I also find that many of my college students benefit from similar activities.

Activity D: Circumference and Diameter of a Circle

Organize the children in pairs. Give them a variety of circular objects (plastic lids, bicycle wheel, circular trash can, etc.) and narrow adding machine tape or paper strips torn from the edges of computer paper. Have them measure the circumference and diameter of the circles by tearing off strips of paper equal to those lengths. For each circle, have them use the diameter strip to measure the circumference strip. Ask how many diameters make a circumference.

Discussion: The activity will help children understand circumference and diameter. They will learn that circumference means *length*. They will also discover that the ratio of the circumference of a circle to its diameter is a little more than 3. This will give them a hands-on understanding of π. By folding their diameter strips, they can arrive at fractions of a diameter and a better estimate of π. Older students can obtain a decimal estimate of π by measuring the strips of paper in centimeters and using a calculator to divide the circumference by the diameter. A fourth-grade teacher used lids and paper strips to make a bulletin board, which was featured at a school open-house.

Activity E: Surface Area and Volume of a Cylinder

Give each pair of students an empty food can and a centimeter ruler. First, have them measure the can and then compute its volume using a calculator and a textbook formula ($v=\pi r^2h$). Next, have them measure the volume of the can by filling it with water (or beans or rice, etc.) and pouring the contents into a graduated cylinder or beaker. Finally, have them compare their answers and discuss sources of error.

Students can use a similar method to investigate surface area. First, they measure the can in centimeters and apply a formula. Next, they wrap the can with centimeter grid paper and count the squares. Finally, they compare the results.

Discussion: Most textbook exercises provide measurements for geometric figures. By contrast, in this activity the students must decide *what* to measure and *how* to measure it. They must then *do* the measuring. Rather than the teacher or textbook telling them whether their answers are correct, they must compute the answer a different way and compare the results. It would be unlikely that the answers would agree exactly, so this creates a natural situation for discussing sources of error in measurement. For example, did they measure the inside or outside dimensions of the can? What was the effect of rounding? How accurately did they measure with the beaker?

Conclusion

When Ms. Rowland and her colleagues first began to teach lessons like these, which require active engagement by their students, they found their classrooms were noisier than usual, and they felt uncomfortable with the new environment. But with time they came to enjoy the lessons as much as their students did and were pleased with what the students learned. Their assessments came mainly from observing the children in action and hearing them talk about the mathematics they were doing, rather than from paper-and-pencil tests.

Give these lessons a try in your classroom. I think you, too, will see that what your students know depends very much on how they come to know it. And they may even gain a new perspective on what mathematics is and what it means to *do* mathematics.

Robert C. Moore is Professor of Mathematics at Southern College of Seventh-day Adventists, Collegedale, Tennessee. Currently he is working with the mathematics education program as a visiting professor in the Department of Mathematical Sciences at Montana State University, Bozeman, Montana. Dr. Moore has worked as a mathematics and science teacher at the academy and college levels, has taught elementary and secondary mathematics methods classes, and has worked with elementary teachers in Georgia and Tennessee in the area of geometry and measurement and number sense.

REFERENCES

1. National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1989); _____, *Professional Standards for Teaching Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1991).

2. National Research Council, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, D.C.: National Academy Press, 1989).

3. National Council of Teachers of Mathematics, 1989, p. 5.

4. National Council of Teachers of Mathematics, 1991, p. 1.