General Physics Lab 12

Radioactive Decay

Objectives:

- To simulate and analyze radioactive decay

Equipment:

- 100 Pennies
- Container for Pennies
- PhET Radioactive Decay Simulator
    - Online Simulator
    - Desktop Simulator

Physical Principles:

Radioactive Decay

Unstable, radioactive nuclei make transitions to other isotopes, emitting alpha, beta, or gamma ray particles. The number of nuclei, ΔN, that decay in a time interval, Δt, is directly proportional to the number, N, of nuclei present in the sample at that instant and is given by

$$R = \frac{\Delta N}{\Delta t} = -\lambda N \, , \qquad (1)$$

where λ is the decay constant for a particular reaction.

The decay constant, λ, can be understood as the probability that a single radioactive nucleus decays if observed for one second (or year or other appropriate unit of time).

The units for λ will be 1/s, 1/yr, etc.

Eq. (1) leads to an exponential decay with time given by,

$$N = N_0 e^{-\lambda t} \, , \qquad (2)$$

where N_0 is the initial number of radioactive nuclei at time, t = 0.

Since a large value of λ indicates high probability of decay, the half-life, $t_{1/2}$, is inversely related to the decay constant by the following equation.

$$t_{1/2} = \frac{\ln(2)}{\lambda} \approx \frac{0.693}{\lambda} \qquad \text{(units of seconds, years, etc.)} \tag{3}$$

Eq. (2) can be linearized by taking the natural log (ln) of both sides to obtain,

$$\ln(N) = -\lambda t + \ln(N_0)\ . \tag{4}$$

Note: Both parts of the lab need to be completed. Work with your lab partner to divide up the work appropriately (ex. one could do Part 1 and the other could do Part 2).

Procedure (Part 1):

Coin-Toss Simulation

In this simulation of radioactivity, a collection of coins will represent a sample of radioactive nuclei. Each trial of randomly flipping the coins will represent a unit of time (second, year, etc.).

1. Begin with 100 pennies and place them in a bucket or other convenient container.
2. Dump the coins out of the container onto a table.
3. Remove the tails and record the number of heads, N_{coins}, remaining.
4. Place the head coins in the container, shake, and dump them out again.
5. Remove the tails and record the number of heads.
6. Continue in this manner 6 times or until four or fewer heads remain.

Analysis (Part 1):

Exponential Graph

According to Eq. (2), a plot of number of heads, N_{coins}, vs. trial number, t (analogous to time), should yield an exponential graph of the form y = a exp(-cx) + b, where

$$a = N_0\ , \qquad \text{(initial \# of coins)} \tag{5}$$

and

$$c = \lambda\ . \qquad \text{(decay constant)} \tag{6}$$

1. Plot the number of heads, N_{coins} (y-axis) vs. trial number, t (x-axis) – this is analogous to plotting the number of radioactive nuclei in a sample vs. time.
2. Apply an exponential/natural exponent curve fit, and record the fit parameters “a” and “c”.

3. Compare the parameter “a” to $N_{0\ theory} = 100$, by calculating the percent error.

$$\%Error = \frac{|100 - a|}{100} \times 100\%$$

4. Use Eq. (3) with the parameter “c” as the measured decay constant, λ_{meas}, to calculate the measured half-life, $t_{1/2\ meas}$.
5. With 2-sided coins, we would expect that after 1 trial, half the coins are heads (and the other half tails). This gives us our expected half-life, $t_{1/2\ theory} = 1$ trial.
6. Compare the measured half-life, $t_{1/2\ meas}$, to the expected half-life, $t_{1/2\ theory}$, by calculating the percent error.

$$\%Error = \frac{|1 - t_{1/2\ meas}|}{1} \times 100\%$$

Linear Graph

According to Eq. (4), a plot of $\ln(N_{coins})$ vs. trial number, t (analogous to time), should yield a linear graph with y-intercept

$$b = \ln(N_0)\ , \qquad \text{(natural log of initial \# of coins)} \qquad (7)$$

and slope

$$m = -\lambda\ . \qquad \text{(negative decay constant)} \qquad (8)$$

1. Plot $\ln(N_{coins})$ (y-axis) vs. trial number, t (x-axis).
2. Perform a linear fit and record the y-intercept, b, and slope, m.
3. Use Eq. (7) to convert y-intercept, b, into the initial number of coins, $N_{0\ linear}$.
4. Compare $N_{0\ linear}$ to $N_{0\ theory} = 100$, by calculating the percent error.

$$\%Error = \frac{|100 - N_{0\ linear}|}{100} \times 100\%$$

5. Use Eq. (8) to convert slope, m, into the measured decay constant, λ_{linear}.
6. Use Eq. (3) with the measured decay constant, λ_{linear}, to calculate the measured half-life, $t_{1/2\ linear}$.
7. Compare the measured half-life, $t_{1/2\ linear}$, to the expected half-life, $t_{1/2\ theory} = 1$ trial, by calculating the percent error.

$$\%Error = \frac{|1 - t_{1/2\ linear}|}{1} \times 100\%$$

Procedure (Part 2):
Carbon-14 Simulation

Access the PhET radioactive decay simulator using one of the following options and familiarize yourself with the operation of the simulator.

Try the online version first and if it does not work, try it on a different browser. It may work on a phone or tablet too, but if you use it on a touchscreen, you will need to be deliberate when pressing the step button, making sure it was actually pressed. If you want to try the desktop application instead, you can do that as well. Just download the program and run it. It may not work for everyone though, which is why we recommend trying the online version first.

Online Simulator:
https://phet.colorado.edu/sims/cheerpj/nuclear-physics/latest/nuclear-physics.html?simulation=radioactive-dating-game

Desktop Simulator:
https://phet.colorado.edu/en/simulation/legacy/radioactive-dating-game

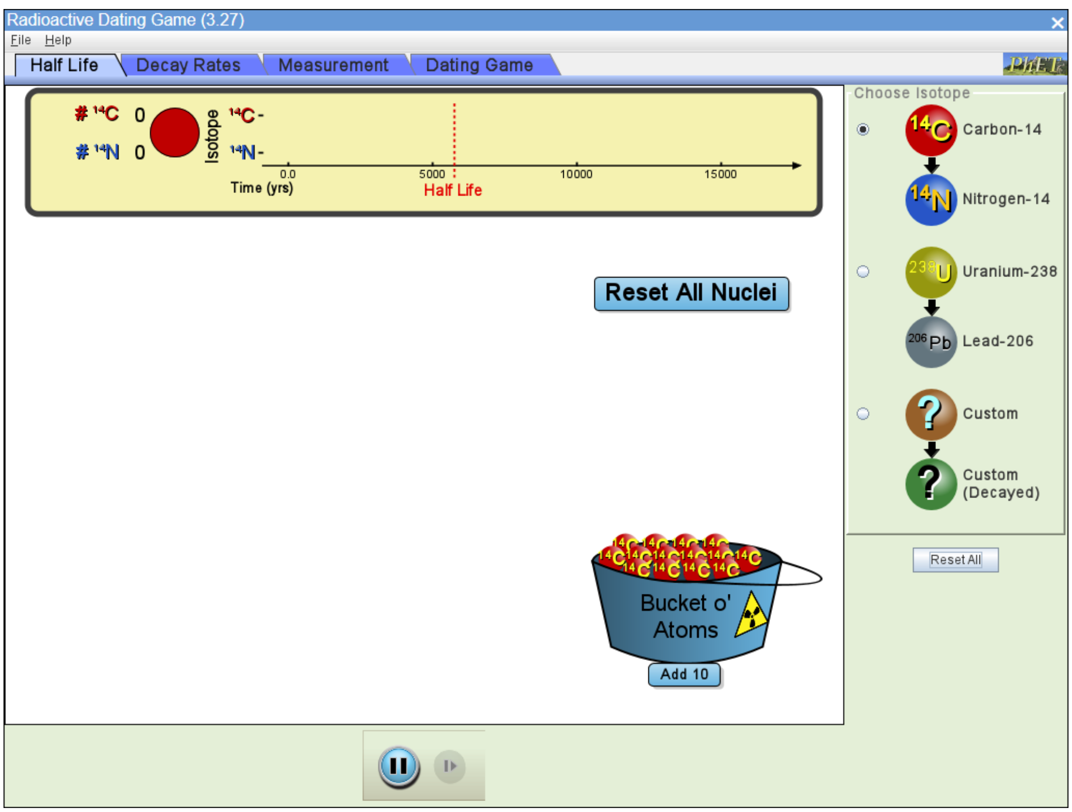

Fig. 1: PhET Radioactive Decay Simulator – Choose "Carbon-14" on the right, click "Reset All", pause the simulation, and add 100 nuclei by clicking "Add 10" ten times. Record $N_0 = 100$ at $t = 0$. Click step ten times, record the time and # of C-14 nuclei, $N_{C\text{-}14}$, and repeat. Stop when # C-14 ≈ 10.

1. On the half-life tab, choose “Carbon-14” as the isotope in the right-hand panel.
2. Click “Reset All” to reset the simulation.
3. Pause the action by hitting the pause button.
4. Add 100 carbon-14 nuclei from the bucket by clicking “Add 10” ten times in a row.
5. Record the initial number of C-14 nuclei, $N_{C\text{-}14} = 100$, at time, t = 0. The simulator may not state this but $N_{C\text{-}14} = N_0 = 100$ at the beginning because you added 100 C-14 nuclei.
6. Each time the step button is pressed, the simulation advances by 150 years. This is too slow a change for recording data, so instead, you will advance by 1500 years.
7. Press the step button (right next to pause) 10 times to advance by 1500 years, then record the time in years (1500 years) and the number of Carbon-14 nuclei, $N_{C\text{-}14}$ (listed at the top next to # ^{14}C). If you use a touchscreen for this, be deliberate when pressing the step button to make sure you know how many times it was pressed.

   Note: You do NOT need to collect any data about nitrogen (^{14}N). The N’s (N_0 & $N_{C\text{-}14}$) listed in this experiment represents the number of nuclei, in this case, Carbon-14 nuclei. Please do NOT confuse these N’s with nitrogen (^{14}N).

8. Press the step button 10 more times to advance by another 1500 years and record the time (3000 years) and number of Carbon-14 nuclei, $N_{C\text{-}14}$.
9. Continue advancing by 1500 year increments and each time record the time in years number of C-14 nuclei (click step 10x, record values, repeat).
10. Stop when the number of C-14 nuclei is around 10.

Analysis (Part 2):

Exponential Graph

According to Eq. (2), a plot of Carbon-14 nuclei, $N_{C\text{-}14}$, vs. time, t, should yield an exponential graph of the form y = a exp(-cx) + b, where

$$a = N_0 \,, \qquad \text{(initial \# of C-14 nuclei)} \tag{9}$$

and

$$c = \lambda \,. \qquad \text{(decay constant)} \tag{10}$$

1. Plot the number of C-14 nuclei, $N_{C\text{-}14}$ (y-axis) vs. time, t (x-axis).
2. Apply an exponential/natural exponent curve fit, and record the fit parameters “a” and “c”.
3. Compare the parameter “a” to $N_{0\,\text{theory}} = 100$, by calculating the percent error.

$$\%Error = \frac{|100 - a|}{100} \times 100\%$$

4. Use Eq. (3) with the parameter "c" as the measured decay constant, λ_{meas}, to calculate the measured half-life, $t_{1/2\ meas}$.
5. Compare the measured half-life, $t_{1/2\ meas}$, to the recognized half-life for Carbon-14, $t_{1/2\ theory}$ = 5730 years, by calculating the percent error.

$$\%Error = \frac{\left|5730 - t_{1/2\ meas}\right|}{5730} \times 100\%$$

Linear Graph

According to Eq. (4), a plot of $\ln(N_{C\text{-}14})$ vs. time, t, should yield a linear graph with y-intercept

$b = \ln(N_0)$, (natural log of initial # of C-14 nuclei) (11)

and slope

$m = -\lambda$. (negative decay constant) (12)

1. Plot $\ln(N_{C\text{-}14})$ (y-axis) vs. time, t (x-axis).
2. Perform a linear fit and record the y-intercept, b, and slope, m.
3. Use Eq. (11) to convert y-intercept, b, into the initial number of C-14 nuclei, $N_{0\ linear}$.
4. Compare $N_{0\ linear}$ to $N_{0\ theory}$ = 100, by calculating the percent error.

$$\%Error = \frac{\left|100 - N_{0\ linear}\right|}{100} \times 100\%$$

5. Use Eq. (12) to convert slope, m into the measured decay constant, λ_{linear}.
6. Use Eq. (3) with the measured decay constant, λ_{linear}, to calculate the measured half-life, $t_{1/2\ linear}$.
7. Compare the measured half-life, $t_{1/2\ linear}$, to the expected half-life, $t_{1/2\ theory}$ = 5730 years, by calculating the percent error.

$$\%Error = \frac{\left|5730 - t_{1/2\ linear}\right|}{5730} \times 100\%$$

Dating Game Simulator

Try out the Dating Game in the PhET simulator. Why is Carbon-14 dating appropriate for once-living objects in upper geologic layers, but uranium dating is appropriate for rocks?