Trigonometric Ratios and Functions

Algebra 2

Chapter 10

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

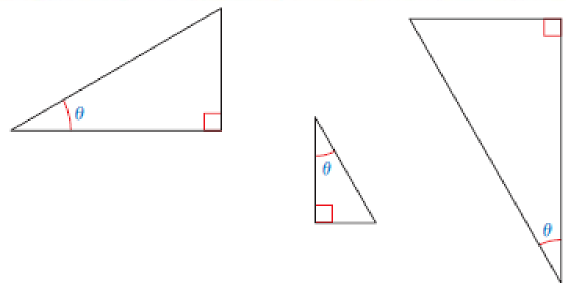
After this lesson…

- I can define the six trigonometric functions.
- I can evaluate trigonometric functions.
- I can use trigonometric functions to find side lengths of right triangles.

10.1 Right Triangle Trigonometry

10.1 Right Triangle Trigonometry

- **Work with a partner.** In each triangle shown, the measure of θ is 30°. Use a centimeter ruler to approximate the following ratios of side lengths. What do you notice?



See page 521 in your textbook

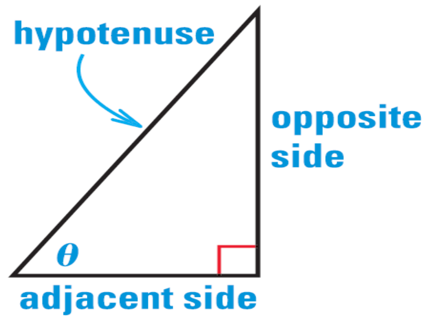
- $\frac{opposite}{hypotenuse}$
- $\frac{adjacent}{hypotenuse}$
- $\frac{opposite}{adjacent}$

10.1 Right Triangle Trigonometry

- If you have a right triangle, there are six ratios of sides that are always constant

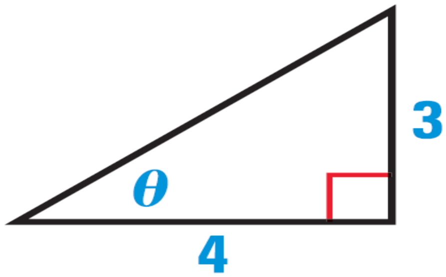
- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
- $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$
- $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
- $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

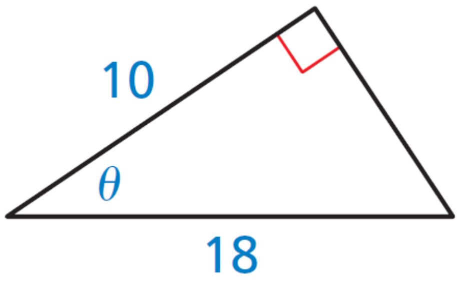
SOH
CAH
TOA



10.1 Right Triangle Trigonometry

- Evaluate the six trigonometric functions of the angle θ.





- Try 526#5

Use Pythagorean Theorem to find hypotenuse

$$3^2 + 4^2 = hyp^2$$
$$hyp = 5$$

$\sin\theta = \frac{3}{5}$ $\quad\cos\theta = \frac{4}{5}$ $\quad\tan\theta = \frac{3}{4}$

$\csc\theta = \frac{5}{3}$ $\quad\sec\theta = \frac{5}{4}$ $\quad\cot\theta = \frac{4}{3}$

Use Pythagorean Theorem to find opposite

$$10^2 + opp^2 = 18^2$$
$$100 + opp^2 = 324$$
$$opp^2 = 224$$
$$opp = 4\sqrt{14}$$

$\sin\theta = \frac{4\sqrt{14}}{18} = \frac{2\sqrt{14}}{9}$ $\quad\cos\theta = \frac{10}{18} = \frac{5}{9}$ $\quad\tan\theta = \frac{4\sqrt{14}}{10} = \frac{2\sqrt{14}}{5}$

$\csc\theta = \frac{9}{2\sqrt{14}} = \frac{9\sqrt{14}}{28}$ $\quad\sec\theta = \frac{9}{5}$ $\quad\cot\theta = \frac{5}{2\sqrt{14}} = \frac{5\sqrt{14}}{28}$

10.1 Right Triangle Trigonometry

- In a right triangle, θ is an acute angle and $\cos\theta = \frac{7}{10}$. What is $\sin\theta$?

Draw triangle and use pythagorean theorem to find opposite side

$$adj = 7$$
$$hyp = 10$$
$$7^2 + \text{opp}^2 = 10^2$$
$$opp^2 = 51$$
$$opp = \sqrt{51}$$
$$\sin\theta = \frac{\sqrt{51}}{10}$$

Draw triangle and use Pythagorean theorem to find adjacent side

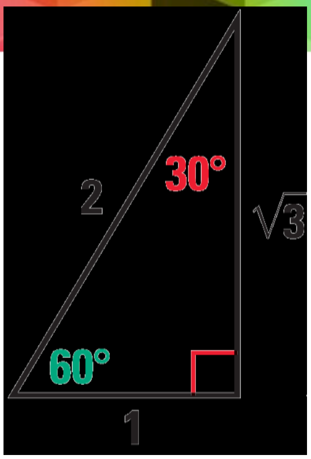
$$opp = 7, hyp = 11$$
$$7^2 + adj^2 = 11^2$$
$$49 + adj^2 = 121$$
$$adj^2 = 72$$
$$adj = 6\sqrt{2}$$

$\sin\theta = \frac{7}{11}$ $\quad$ $\cos\theta = \frac{6\sqrt{2}}{11}$ $\quad$ $\tan\theta = \frac{7}{6\sqrt{2}} = \frac{7\sqrt{2}}{12}$

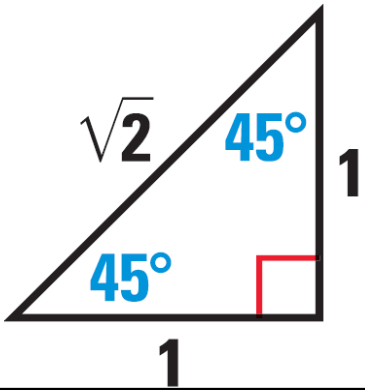
$\csc\theta = \frac{11}{7}$ $\quad$ $\sec\theta = \frac{11}{6\sqrt{2}} = \frac{11\sqrt{2}}{12}$ $\quad$ $\cot\theta = \frac{6\sqrt{2}}{7}$

10.1 Right Triangle Trigonometry

- Special Right Triangles
- 30° - 60° - 90°



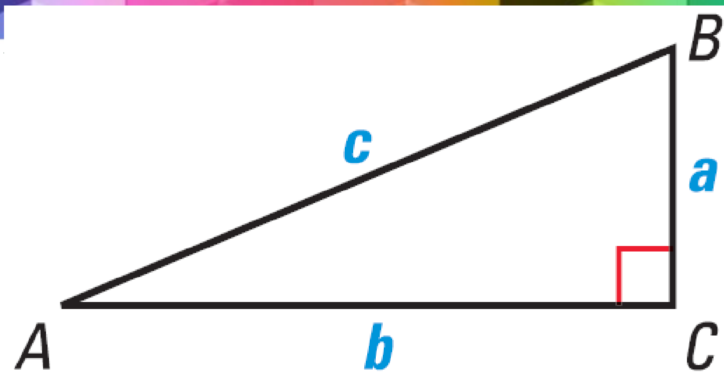
- 45° - 45° - 90°



These triangles can be used to find sin, cos, tan of 30, 60, and 45 degree angles exactly.

10.1 Right Triangle Trigonometry

- Use the diagram to solve the right
- $B = 60°$, $a = 7$



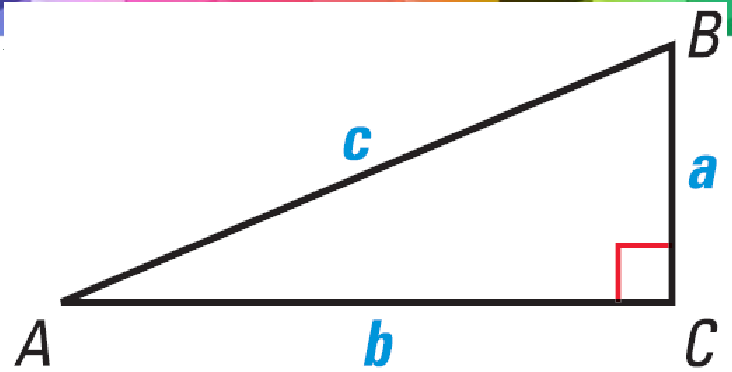
A = 90° - 60° = 30°

From special rt triangle; $\tan 60° = \frac{b}{7} = \frac{\sqrt{3}}{1} \rightarrow b = 7\sqrt{3}$

$\cos 60° = \frac{7}{c} = \frac{1}{2} \rightarrow c = 14$

10.1 Right Triangle Trigonometry

- Use the diagram to solve the right
- $A = 32°$, $b = 10$



- Try 526#33
  $A = 43°$, $b = 31$

B = 90° - 32° = 58°

$\tan 32° = \frac{a}{10} \rightarrow a = 10 \tan 32° \approx 6.25$

$\cos 32° = \frac{10}{c} \rightarrow c \cdot \cos 32° = 10 \rightarrow c = \frac{10}{\cos 32°} \approx 11.79$

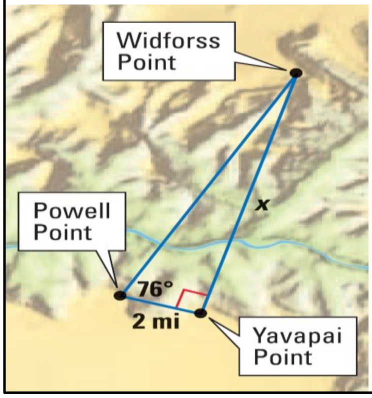
#33: B = 90° - 43° = 47°

$\tan 43° = \frac{a}{31} \rightarrow a = 31 \tan 43° \approx 28.91$

$\cos 43° = \frac{31}{c} \rightarrow c \cdot \cos 43° = 31 \rightarrow c = \frac{31}{\cos 43°} \approx 42.39$

10.1 Right Triangle Trigonometry

- Find the distance between Powell Point and Widforss Point.



$$\cos 76° = \frac{2\text{ mi}}{y}$$
$$y \cdot \cos 76° = 2\text{ mi}$$
$$y = \frac{2\text{ mi}}{\cos 76°} \approx 8.27\text{ mi}$$

#37:

$$\tan 79° = \frac{w}{100}$$
$$w = 100\tan 79° \approx 514\, m$$

After this lesson…

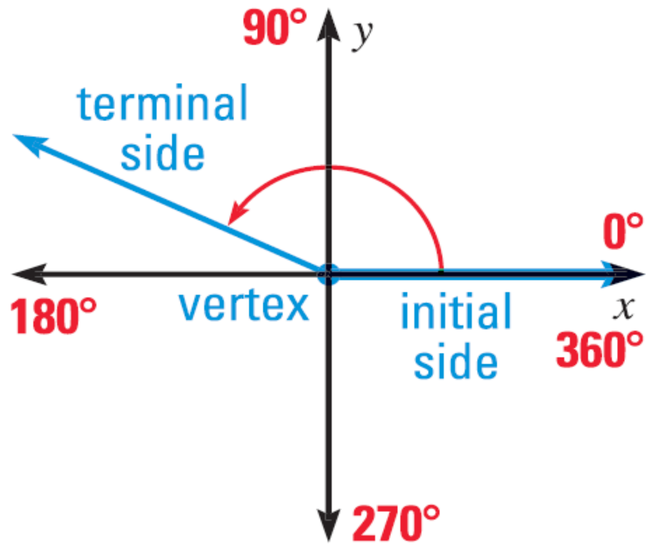
- I can draw angles in standard position.
- I can explain the meaning of radian measure.
- I can convert between degrees and radians.
- I can use radian measure to find arc lengths and the area of a sector.

10.2 Angles and Radian Measure

12

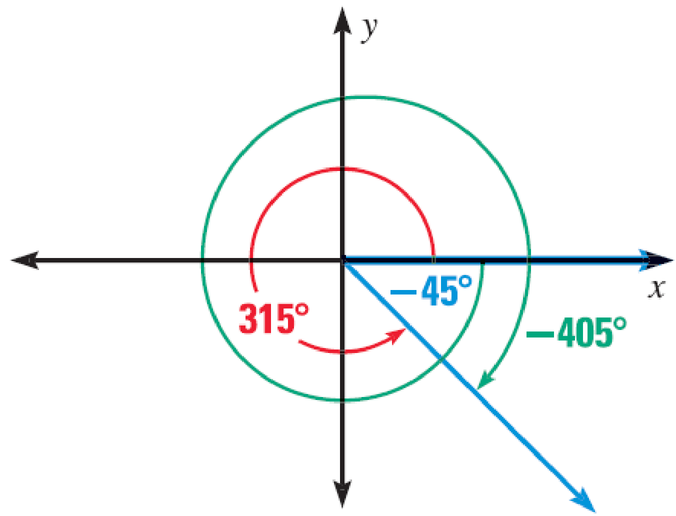
10.2 Angles and Radian Measure

- Angles in Standard Position
  - Vertex on origin
  - Initial Side on positive x-axis
  - Measured counterclockwise



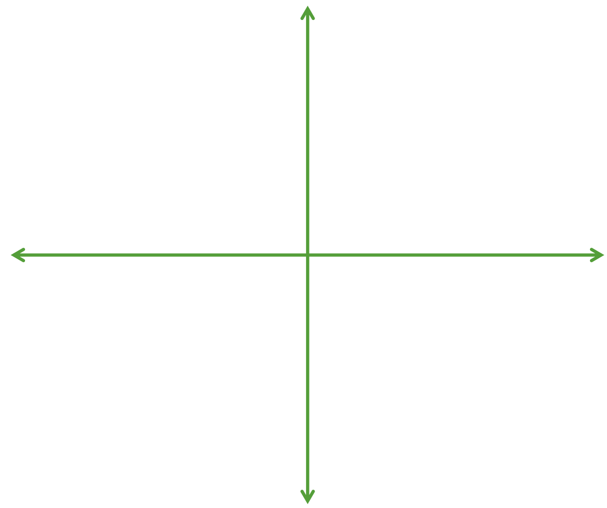
10.2 Angles and Radian Measure

- Coterminal Angles
  - Different angles (measures) that have the same terminal side
  - Found by adding or subtracting multiples of 360°



10.2 Angles and Radian Measure

- Draw an angle with the given measure in standard position. Then find one positive coterminal angle and one negative coterminal angle.
- 65°
- −900°
- 300°
- Try 534#7: −125°



65°+360°=425°; 65°-360°=-295°

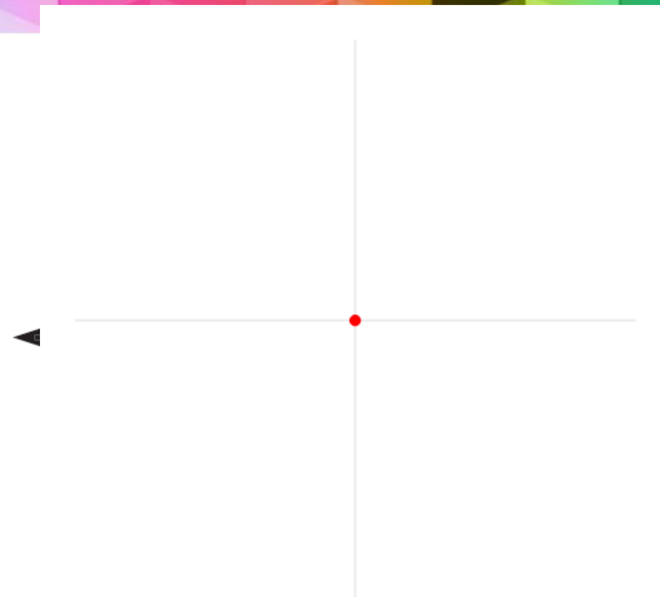
#3: −900°+360°=−540°; −540°+360°=−180°; −180°+360°=180°

300°+360°=660°; 300°-360°=-60°

#5: −125°+360°=235°; −125°−360°=−485°

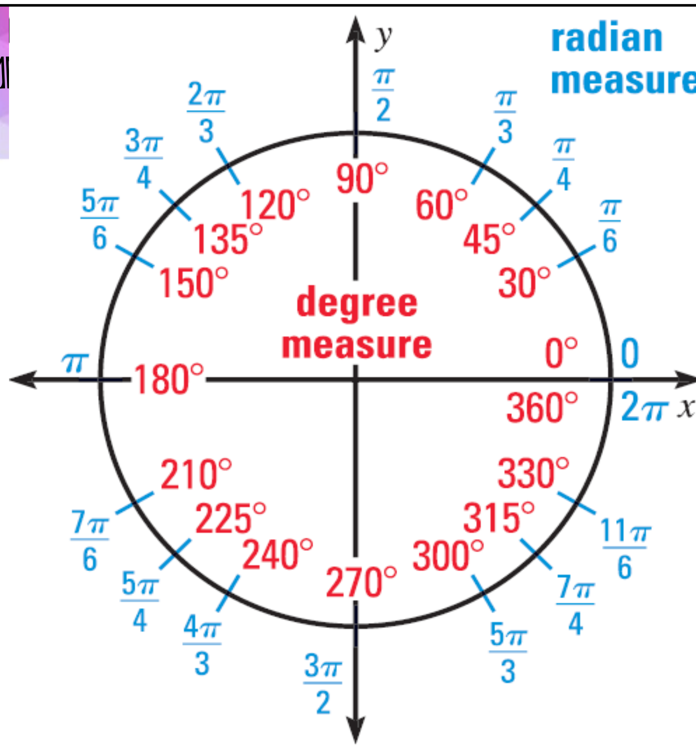
10.2 Angles and Radian Measure

- Radian measure
  - Another unit to measure angles
  - 1 radian is the angle when the arc length = the radius
  - There are 2π radians in a circle



10.2 Angles and Radian M

- To convert between degrees and radians use fact that
  - 180° = π
- Special angles



Probably only need to memorize 1st quadrant

10.2 Angles and Radian Measure

- Convert the degree measure to radians, or the radian measure to degrees.
- 135°
- $\frac{5\pi}{4}$
- −50°
- Try 534#9
  40°

$$135°\left(\frac{\pi}{180}°\right) = \frac{3\pi}{4}$$

$$\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$$

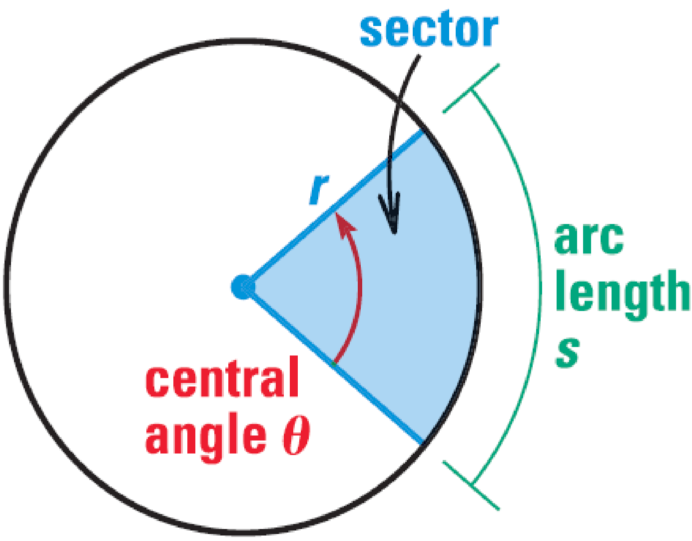
$$-50°\left(\frac{\pi}{180}°\right) = -\frac{5\pi}{18}$$

#9

$$40°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{9}$$

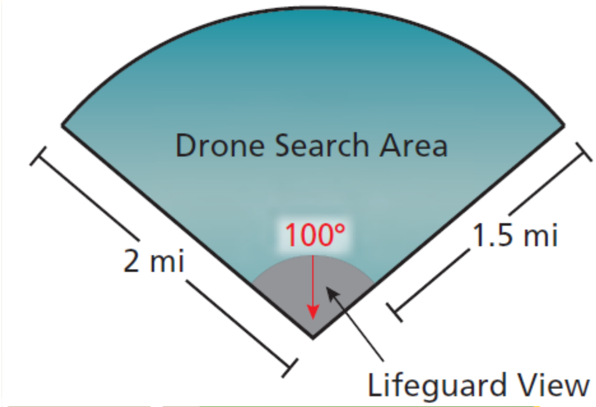
10.2 Angles and Radian Measure

- Sector
  - Slice of a circle
- Arc Length
  - $s = r\theta$
  - θ must be in radians!
- Area of Sector
  - $A = \frac{1}{2}r^2\theta$
  - θ must be in radians!



10.2 Angles and Radian Measure

- Find the length of the outfield fence if it is 220 ft from home plate.
- Find the area of the baseball field.
- Try 535#29 Find the Drone Search Area



$$\theta = \frac{\pi}{2}$$
$$s = r\theta$$
$$s = 220\left(\frac{\pi}{2}\right) = 110\pi \approx 346$$

$$A = \frac{1}{2}r^2\theta$$
$$A = \frac{1}{2}(220)^2\left(\frac{\pi}{2}\right) = 12100\pi \approx 38013$$

#29

Convert to radians. $100°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{9}$

$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\theta$$
$$A = \frac{1}{2}(2)^2\left(\frac{5\pi}{9}\right) - \frac{1}{2}(1.5)^2\left(\frac{5\pi}{9}\right) \approx 1.53\ mi^2$$

After this lesson…

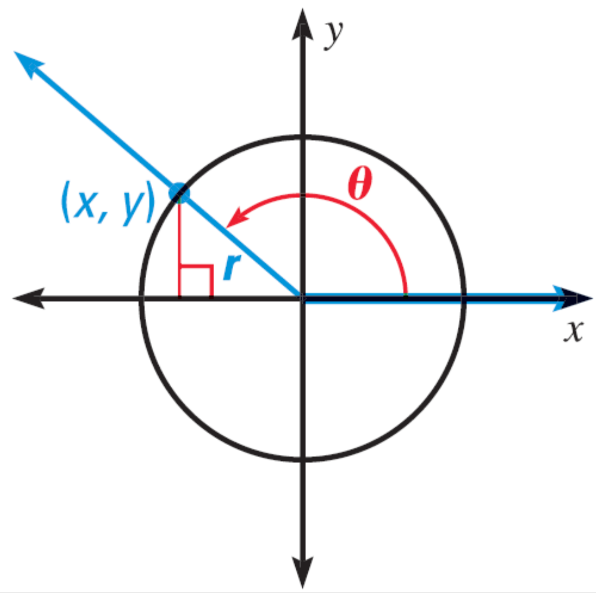
- I can evaluate trigonometric functions given a point on an angle.
- I can evaluate trigonometric functions using the unit circle.
- I can find and use reference angles to evaluate trigonometric functions.
- I can solve real-life problems involving projectiles.

10.3 Trigonometric Functions of Any Angle

21

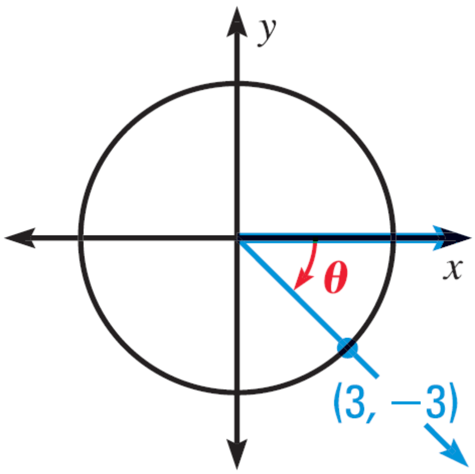
10.3 Trigonometric Functions of Any Angle

- Think of a point on the terminal side of an angle
- You can draw a right triangle with the *x*-axis
- $\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$
- $\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$
- $\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$
- Unit Circle
  - $r = 1$



10.3 Trigonometric Functions of Any Angle

- Evaluate the six trigonometric functions of θ.



$$r^2 = x^2 + y^2 = 3^2 + (-3)^2$$
$$r = \sqrt{18} = 3\sqrt{2}$$

$\sin\theta = -\frac{3}{3\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\cos\theta = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\tan\theta = -\frac{3}{3} = -1$

$\csc\theta = -\sqrt{2}$ $\sec\theta = \sqrt{2}$ $\cot\theta = -1$

10.3 Trigonometric Functions of Any Angle

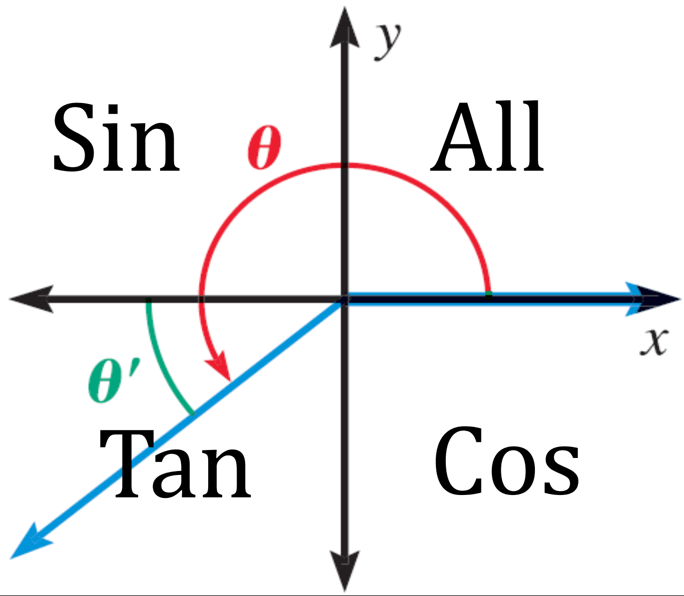
- Quadrantal Angles
- Evaluate the six trigonometric functions of θ.
- θ = 180°

$\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$

$\csc\theta = und$ $\sec\theta = -1$ $\cot\theta = und$

10.3 Trigonometric Functions of Any Angle

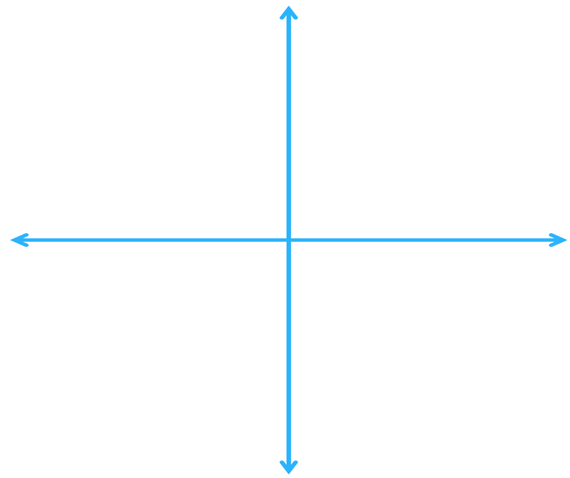
- Reference Angle
  - Angle between terminal side and *x*-axis
  - Has the same values for trig functions as 1st quadrant angles
  - You just have to add the negative signs



This gives what is positive.
The reciprocal functions are the same (csc is with sin, etc.)
Way to remember "All Students Take Calculus"

10.3 Trigonometric Functions of Any Angle

- Sketch the angle. Then find its reference angle.
- $150°$
- Try 542#15 $\frac{23\pi}{4}$



$$180° - 150° = 30°$$

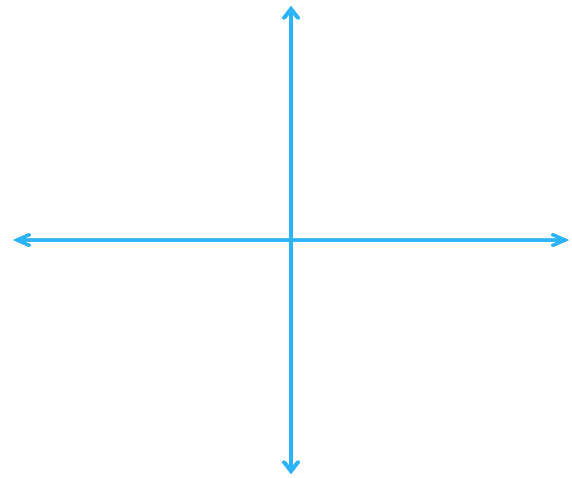
#15

$$\frac{23\pi}{4} - 2\pi = \frac{15\pi}{4} \rightarrow \frac{15\pi}{4} - 2\pi = \frac{7\pi}{4}$$
$$2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$$

10.3 Trigonometric Functions of Any Angle

- Evaluate cos(-60°) without a calculator

- Try 542#25
  $\sin(-150°)$



Reference angle is 60°
In quadrant IV (cos positive)
Use 30°-60°-90° triangle

$$\cos(-60°) = \frac{1}{2}$$

#25
Reference angle is 30°
In quadrant III (sin negative)
Use 30°-60°-90°

$$\sin(-150°) = -\sin(30°) = -\frac{1}{2}$$

10.3 Trigonometric Functions of Any Angle

- Estimate the horizontal distance traveled by a Red Kangaroo who jumps at an angle of 8° and with an initial speed of 53 feet per second (35 mph).

$$d = \frac{v^2}{32}\sin 2\theta$$

$$d = \frac{53^2}{32}\sin\left(2(8°)\right) = 24.2\ ft$$

After this lesson…

- I can identify characteristics of sine and cosine functions.
- I can graph transformations of sine and cosine functions.

10.4 Graphing Sine and Cosine Functions

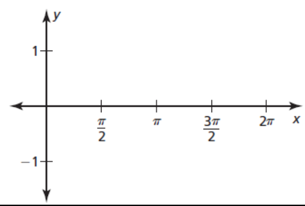
30

10.4 Graphing Sine and Cosine Functions

- **Work with a partner.**
- **a.** Complete the table for $y = \sin x$, where x is an angle measure in radians.

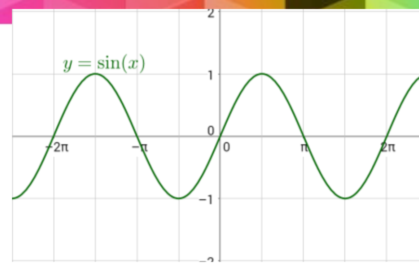
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$y = \sin x$									

- **b.** Plot the points (x, y) from part (a). Draw a smooth curve through the points to sketch the graph of $y = \sin x$. Make several observations about the graph.

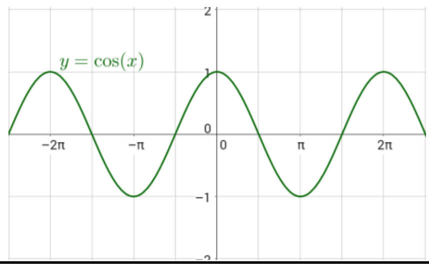


10.4 Graphing Sine and Cosine Functions

- $y = \sin x$
  - Starts at 0
  - Amplitude = 1
  - Period = 2π



- $y = \cos x$
  - Starts at 1
  - Amplitude = 1
  - Period = 2π



Point out
- Amplitude
- period
- key points

10.4 Graphing Sine and Cosine Functions

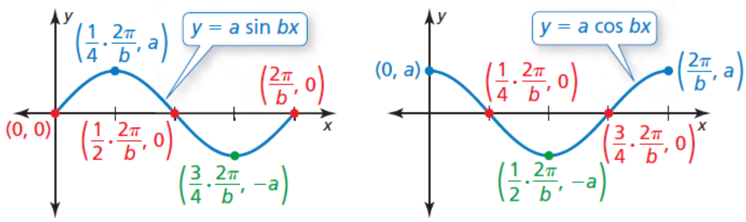
- Transformations
- $y = a \sin b(x - h) + k$
  - $|a|$ = amplitude = vertical stretch
    - If $a < 0$, then reflect over x-axis
  - b = horizontal shrink
    - Period $T = \frac{2\pi}{|b|}$
  - h = horizontal shift
  - k = vertical shift
    - Midline $y = k$

c is like h
d is like k

10.4 Graphing Sine and Cosine Functions

- Graphing sine and cosine

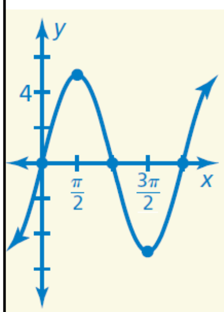
1. Identify the amplitude, period, horizontal shift, and vertical shift
2. Draw the midline, $y = k$
3. Find the 5 key points (3 zeros, 1 max, 1 min)



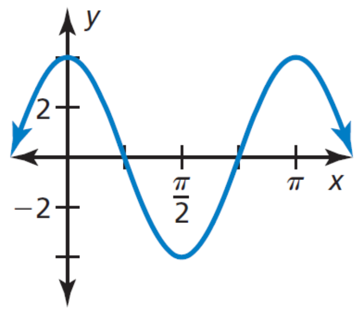
4. Draw the graph

34

- Identify the amplitude and period.



- Try 551#7

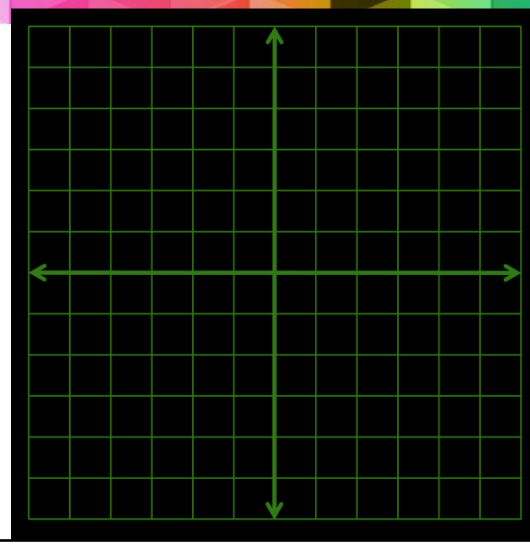


35

amp: 5; period: 2π
#7
Amp: 4; period: π

10.4 Graphing Sine and Cosine Functions

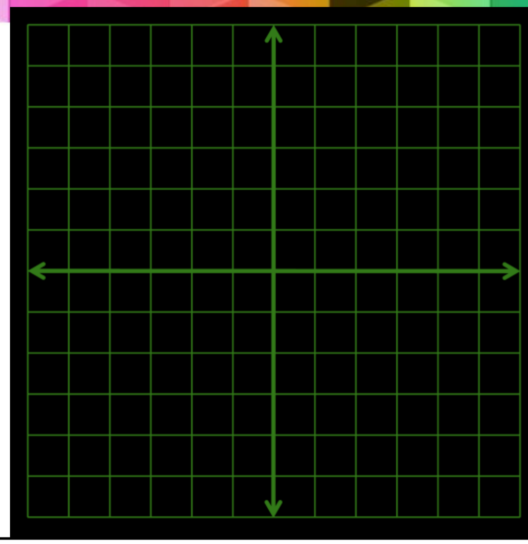
- Graph $f(x) = 2\sin x$



Same as sine, but amp = 2

10.4 Graphing Sine and Cosine Functions

- Try 551#13
  Graph $y = \sin 2\pi x$



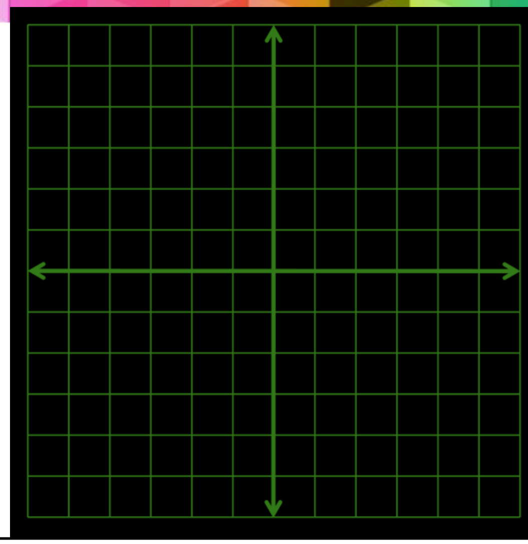
Period $T = \frac{2\pi}{b}$

$$T = \frac{2\pi}{2\pi} = 1$$

Amp = 1

10.4 Graphing Sine and Cosine Functions

- Graph $y = 2\cos\left(x - \frac{\pi}{2}\right)$

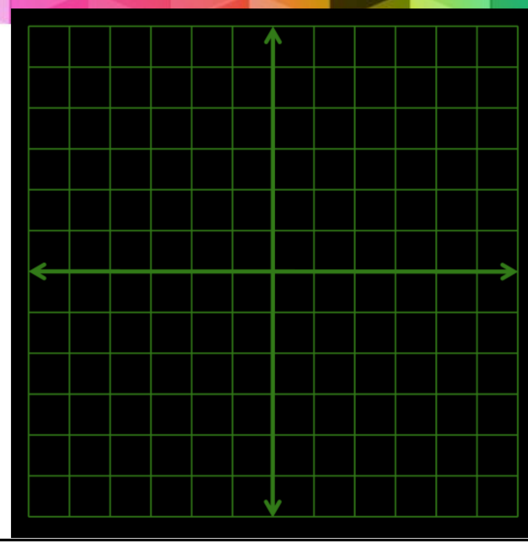


$$a = 2$$
$$b = 1 \ \rightarrow T = 2\pi$$
$$h = \frac{\pi}{2}\ to\ right$$

Draw $2\cos x$ first and then do the phase shift

10.4 Graphing Sine and Cosine Functions

- Try 552#39
  Graph
  $$g(x) = -4\cos\left(x + \frac{\pi}{4}\right) - 1$$



$$a = -4 = amp$$

$$b = 1 \to T = \frac{2\pi}{b} \to \frac{2\pi}{1} = 2\pi$$

$$h = -\frac{\pi}{4}$$

Shift left π/4

$$k = -1$$

Shift down 1
Graph $-4\sin x$ first labeling the key points with a period of 2π
Reflect over the *x*-axis because *a* is negative
Shift left π/4 and down 1

After this lesson…

- I can identify characteristics of tangent, cotangent, secant, and cosecant functions.
- I can graph tangent and cotangent functions.
- I can graph secant and cosecant functions.

10.5 Graphing Other Trigonometric Functions

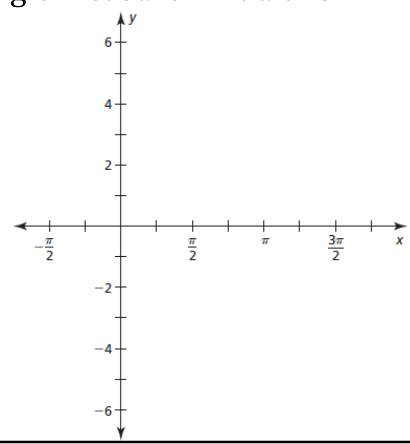
40

10.5 Graphing Other Trigonometric Functions

- **Work with a partner.**
- **a.** Complete the table for $y = \tan x$, where x is an angle measure in radians.

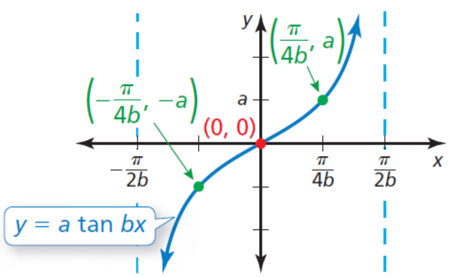
x	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$
$y = \tan x$								
x	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$y = \tan x$								

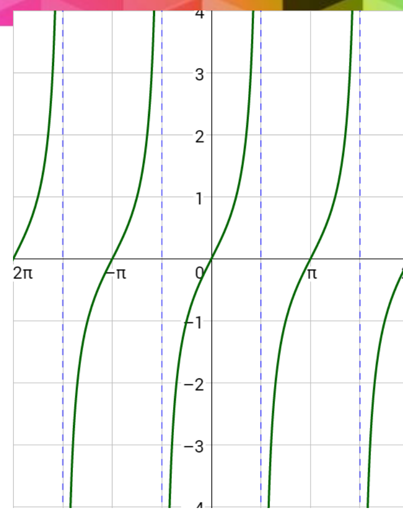
- **b.** Plot the points (x, y) from part (a). Then sketch the graph of $y = \tan x$. Make several observations about the graph.



10.5 Graphing Other Trigonometric Functions

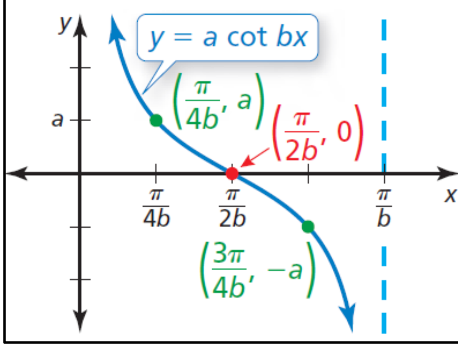
- $y = \tan x$
  - Period = π
    - $T = \frac{\pi}{b}$
  - Asymptotes where tangent undefined, $\frac{\pi}{2}, \frac{3\pi}{2}$

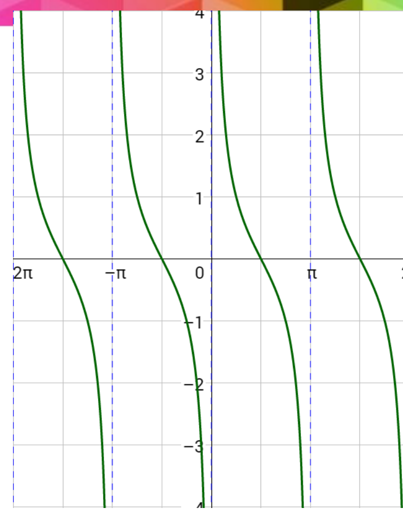




10.5 Graphing Other Trigonometric Functions

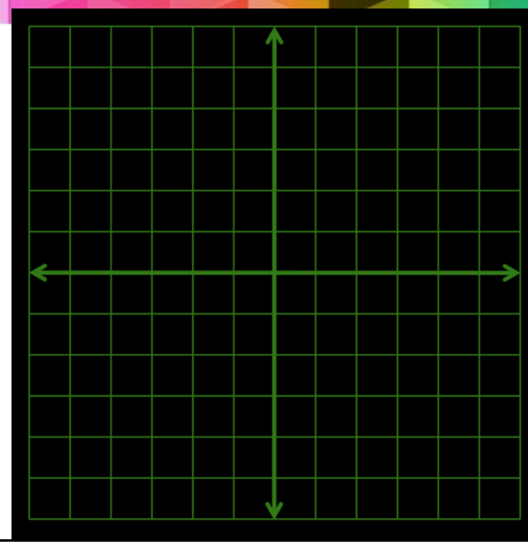
- $y = \cot x$
  - Period = π
    - $T = \frac{\pi}{b}$
  - Asymptotes at 0, π, 2π





10.5 Graphing Other Trigonometric Functions

- Graph $y = \tan\frac{x}{4}$



$$b = \frac{1}{4}$$

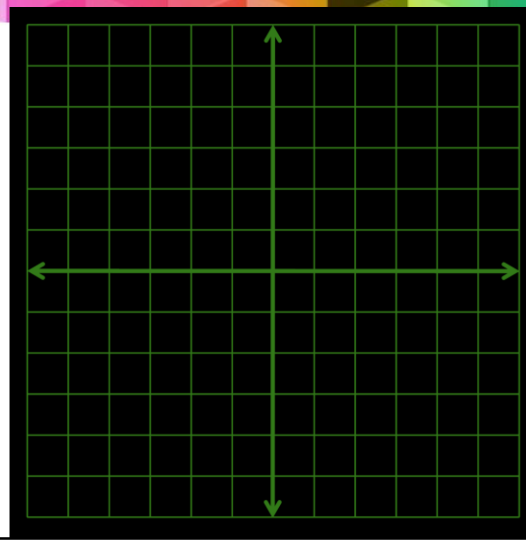
$$T = \frac{\pi}{b} = \frac{\pi}{\frac{1}{4}} = 4\pi$$

$$a = 1$$

10.5 Graphing Other Trigonometric Functions

- Try 560#7

  Graph $g(x) = \frac{1}{2}\tan \pi x$



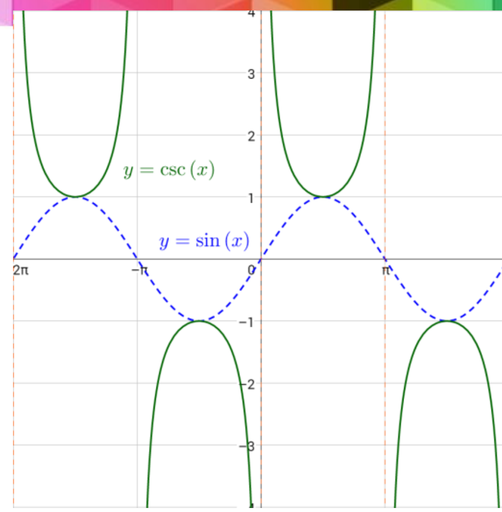
$$b = \pi$$

$$T = \frac{\pi}{b} = \frac{\pi}{\pi} = 1$$

$$a = \frac{1}{2}$$

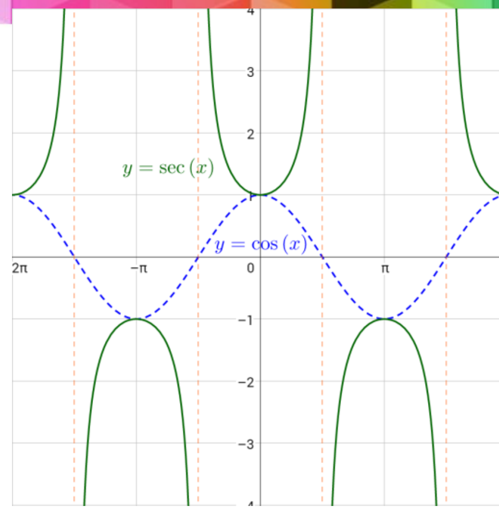
10.5 Graphing Other Trigonometric Functions

- $y = \csc x$
  - Period = 2π
    - $T = \frac{2\pi}{b}$
  - Asymptotes where sine = 0
    - $0, \pi, 2\pi$



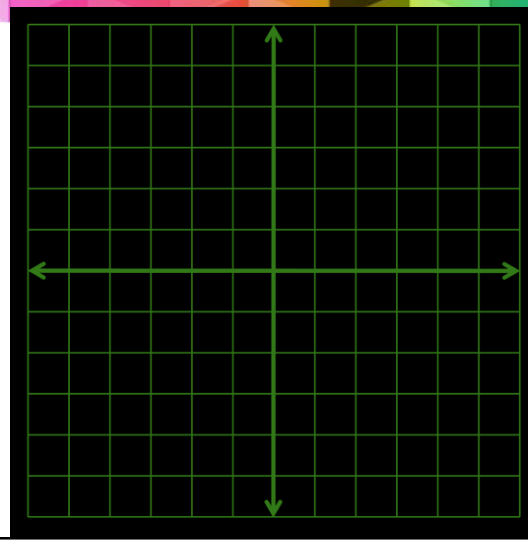
10.5 Graphing Other Trigonometric Functions

- $y = \sec x$
  - Period = 2π
    - $T = \frac{2\pi}{b}$
  - Asymptotes where cosine = 0
    - $\frac{\pi}{2}, \frac{3\pi}{2}$



10.5 Graphing Other Trigonometric Functions

- Graph $y = 2\csc\left(x + \frac{\pi}{2}\right)$

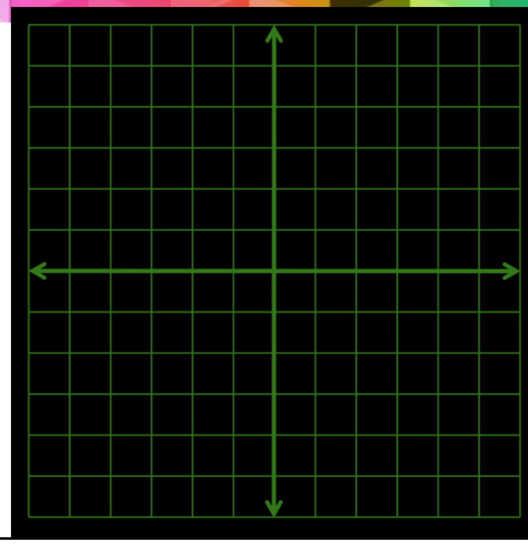


$$a = 2$$
$$b = 1$$
$$T = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$
$$c = -\frac{\pi}{2}$$
$$Horizontal\ shift; h = -\frac{\pi}{2}$$
$$k = 0$$

Start by graphing $2\sin x$
Then shift left $\frac{\pi}{2}$
Then draw asymptotes at the x-intercepts
Then draw csc graph

10.5 Graphing Other Trigonometric Functions

- Try 560#15
  Graph $g(x) = \csc 4x$



$$a = 1$$
$$b = 4$$
$$T = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Start by graphing $\sin 4x$
Then draw asymptotes at the x-intercepts
Then draw csc graph

After this lesson…

- I can write and graph trigonometric functions using frequency.
- I can write trigonometric functions for a given graph.
- I can find a trigonometric model for a set of data using technology.

10.6 Modeling with Trigonometric Functions

10.6 Modeling with Trigonometric Functions

- Trigonometric functions are periodic
  - Useful for modeling oscillating motions or repeated patterns

- Period (T)
  - Time of 1 cycles
    - Unit: s/cycle = s

- Frequency (f)
  - Cycles per 1 second
    - Unit: cycles/s = Hz

$$T = \frac{1}{f}$$

51

Period and frequency are reciprocals

10.6 Modeling with Trigonometric Functions

- Find the frequency

$$y = 2\cos 3x$$

- Try 568#3

$$y = \sin 3\pi x$$

52

Find period.

$$T = \frac{2\pi}{b} = \frac{2\pi}{3}$$

$$f = \frac{1}{T} = \frac{1}{\frac{2\pi}{3}} = \frac{3}{2\pi}$$

#3

$$T = \frac{2\pi}{b} = \frac{2\pi}{3\pi} = \frac{2}{3}$$

$$f = \frac{1}{T} = \frac{3}{2}$$

10.6 Modeling with Trigonometric Functions

- Write Trigonometric Models
- Find the midline (average of max and min)
- Find the amplitude
- Find the period
- If the situation starts at zero, use sine
  - If starts increasing +
  - If starts decreasing –
- If the situation starts at a maximum or minimum use cosine
  - If starts at max +
  - If starts at min –

10.6 Modeling with Trigonometric Functions

- An audiometer produces a pure tone with a frequency f of 1000 hertz (cycles per second). The maximum pressure P produced by the tone is 20 millipascals. Write a sine model that gives the pressure P as a function of the time t (in seconds).

54

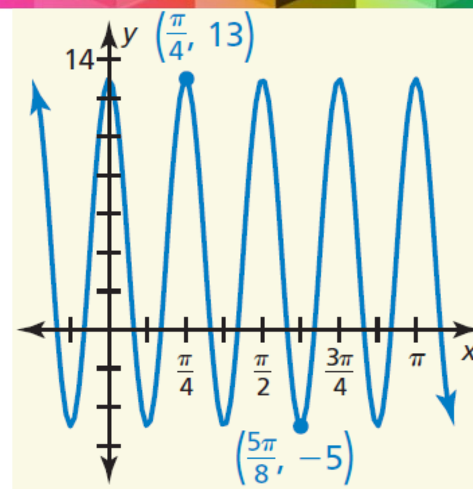
Sound is produced by compressing air

$$f = 1000\ Hz$$
$$T = \frac{1}{f} = \frac{2\pi}{b} = \frac{1}{1000} \rightarrow b = 2000\pi$$
$$a = 20\ mPa$$

$$P = 20 \sin 2000\pi t$$

10.6 Modeling with Trigonometric Functions

- Write a function for the sinusoid shown.



56

y-intercept is a max, so use cosine and *h* = 0

Midline is $y = \frac{13+(-5)}{2} = 4 = k$

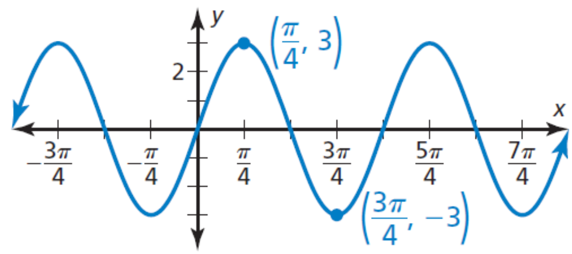
Amplitude is $13 - 4 = 9 = a$

Period is $T = \frac{\pi}{4} = \frac{2\pi}{b} \rightarrow \pi b = 8\pi \rightarrow b = 8$

$$y = a\cos b(x-h) + k$$
$$y = 9\cos 8(x-0) + 4$$
$$y = 9\cos 8x + 4$$

10.6 Modeling with Trigonometric Functions

- Try 569#11
  Write a function for the sinusoid shown.



57

y-intercept is a 0, so use sine and h = 0
Midline is $y = 0 = k$
Amplitude is $3 - 0 = 3 = a$
Period is $T = \pi = \frac{2\pi}{b} \rightarrow \pi b = 2\pi \rightarrow b = 2$

$$y = a \sin b(x - h) + k$$
$$y = 3 \sin 2(x - 0) + 0$$
$$y = 3 \sin 2x$$

10.6 Modeling with Trigonometric Functions

- Two people swing jump ropes. The highest point of the middle of each rope is 80 inches above the ground and the lowest point is 2 inches above the ground. Each rope makes 2 revolutions per second. Write a model for the height *h* (in inches) of one of the ropes as a function of the time *t* (in seconds) given that the rope is at its lowest point when *t* = 0.

Midline: $\frac{80+2}{2} = 41 = k$

Amplitude: $80 - 41 = 39 = a$

Frequency: $f = 2 = \frac{1}{T} \rightarrow T = \frac{1}{2} = \frac{2\pi}{b} \rightarrow b = 4\pi$

Lowest point at *t* = 0, so use cosine with −*a*, $a = -39$

$$y = a \cos b(x - h) + k$$
$$h(t) = -39 \cos 4\pi t + 41$$

10.6 Modeling with Trigonometric Functions

- The tables show the average monthly low temperatures D (in degrees Fahrenheit) in Erie, Pennsylvania, where $t = 1$ represents January. Write a model that gives D as a function of t and interpret the period of its graph. Use technology.

- Press STAT, Edit… enter points
- Press STAT → CALC, SinReg

t	D
1	21
2	21
3	28
4	38
5	48
6	58

t	D
7	64
8	62
9	56
10	45
11	37
12	27

Use a graphing calculator

$D = 21.3 \sin(0.52t - 2.3) + 42$

The period 12 makes sense because there are 12 months in a year, and you expect this pattern to continue in the years to follow.

After this lesson…

- I can evaluate trigonometric functions using trigonometric identities.
- I can simplify trigonometric expressions using trigonometric identities.
- I can verify trigonometric identities.

10.7 Using Trigonometric Identities

61

10.7 Using Trigonometric Identities

- On unit circle
  - $x = \cos\theta$
  - $y = \sin\theta$
- $x^2 + y^2 = 1$
- $\cos^2\theta + \sin^2\theta = 1$

- Trigonometric Identity
- Statement showing relationship between two quantities that are always =

10.7 Using Trigonometric Identities

- Reciprocal Identities
- $\sin u = \frac{1}{\csc u}$ $\csc u = \frac{1}{\sin u}$
- $\cos u = \frac{1}{\sec u}$ $\sec u = \frac{1}{\cos u}$
- $\tan u = \frac{1}{\cot u}$ $\cot u = \frac{1}{\tan u}$

- Quotient Identities
- $\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$

- Pythagorean Identities
- $\sin^2 u + \cos^2 u = 1$
- $\tan^2 u + 1 = \sec^2 u$
- $1 + \cot^2 u = \csc^2 u$

Colored ones should be memorized.

10.7 Using Trigonometric Identities

- Even/Odd Identities
- $\cos(-u) = \cos u$
- $\sec(-u) = \sec u$
- $\sin(-u) = -\sin u$
- $\tan(-u) = -\tan u$
- $\csc(-u) = -\csc u$
- $\cot(-u) = -\cot u$

- Cofunction Identities
- $\sin\left(\frac{\pi}{2} - u\right) = \cos u$
- $\cos\left(\frac{\pi}{2} - u\right) = \sin u$
- $\tan\left(\frac{\pi}{2} - u\right) = \cot u$
- $\cot\left(\frac{\pi}{2} - u\right) = \tan u$
- $\sec\left(\frac{\pi}{2} - u\right) = \csc u$
- $\csc\left(\frac{\pi}{2} - u\right) = \sec u$

10.7 Using Trigonometric Identities

- Given that $\sin\theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of the other five trigonometric functions of θ.

65

Quadrant III, so $x<0$ and $y<0$

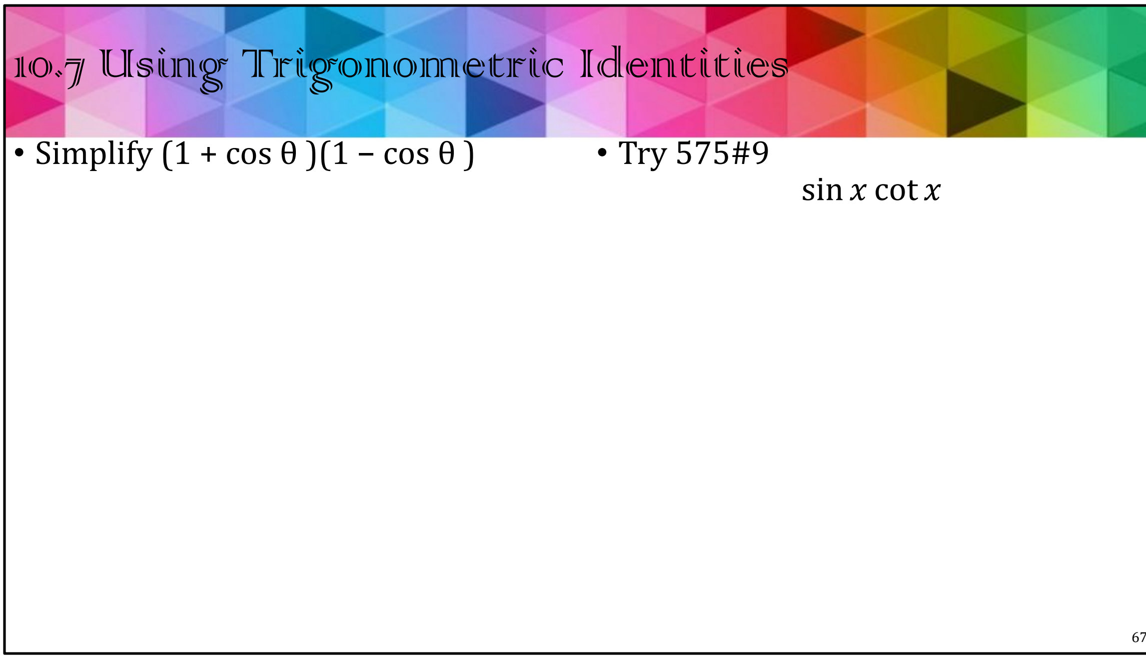
$$\sin^2\theta + \cos^2\theta = 1 \to \left(-\frac{5}{13}\right)^2 + \cos^2\theta = 1 \to \cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169} \to \cos\theta = -\frac{12}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{5}{13}}{-\frac{12}{13}} = \frac{5}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{13}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{13}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$$



$$(1+\cos\theta)(1-\cos\theta)$$
$$1-\cos^2\theta$$
(because $\sin^2\theta+\cos^2\theta=1$, then $\cos^2\theta=1-\sin^2\theta$)
$$1-(1-\sin^2\theta)$$
$$\sin^2\theta$$

#9

$$\sin x\cot x$$
$$\sin x\left(\frac{\cos x}{\sin x}\right)$$
$$\cos x$$

10.7 Using Trigonometric Identities

- Verify Trigonometric Identities
- Show that trig identities are true by turning one side into the other side

- Guidelines

1. Work with 1 side at a time. Start with the more complicated side.
2. Try factor, add fractions, square a binomial, etc.
3. Use fundamental identities
4. If the above doesn't work, try rewriting in sines and cosines
5. Try something!

10.7 Using Trigonometric Identities

- Verify $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

- Try 575#21

$$\cos\left(\frac{\pi}{2} - x\right)\cot x = \cos x$$

69

$$\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$\frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}} = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

#21

$$\cos\left(\frac{\pi}{2} - x\right)\cot x$$

$$\sin x \cot x$$

$$\sin x \left(\frac{\cos x}{\sin x}\right)$$

$$\cos x$$