PROPERTIES OF TRANSFORMATIONS

Geometry

Chapter 4

Geometry 4

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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After this lesson…

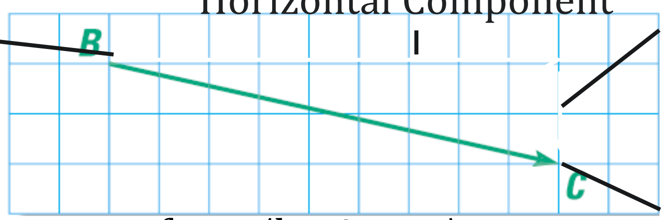
- I can translate figures.
- I can write a translation rule for a given translation.
- I can explain what a rigid motion is.
- I can perform a composition of translations on a figure.

4.1 TRANSLATIONS

4.1 Translations

4.1 Translations

- Vector ($\overrightarrow{BC}$)
  - Measurement with Direction and Magnitude (size)
  - Represented by an arrow

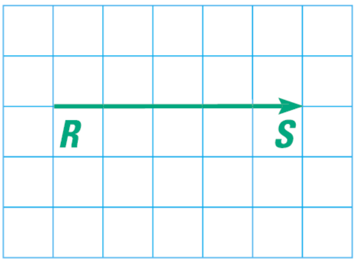


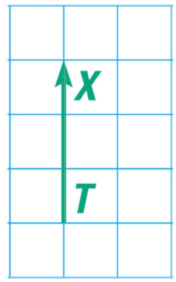
  - Component form ⟨hori, vert⟩
    - $\overrightarrow{BC} = \langle 9, -2 \rangle$

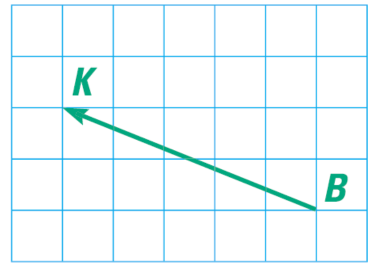
Can be used to describe translations

4.1 Translations

- Name the vector and write its component form







$$\overrightarrow{RS} = \langle 5,0 \rangle$$
$$\overrightarrow{TX} = \langle 0,3 \rangle$$
$$\overrightarrow{BK} = \langle -5,2 \rangle$$

4.1 Translations

- Transformation
  - Moves or changes a figure
  - Original called preimage (i.e. ΔABC)
  - New called image (i.e. $\Delta A'B'C'$)

- Translation
  - Moves every point the same distance in the same direction
  - $(x, y) \rightarrow (x + a, y + b)$
    - where the $\langle a, b \rangle$ is the translation vector

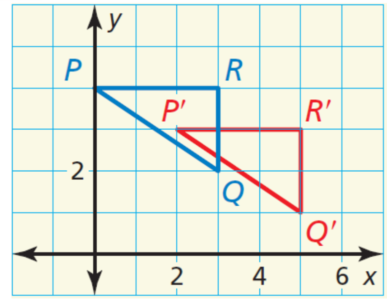
4.1 Translations

- The vertices of ΔLMN are $L(2, 2)$, $M(5, 3)$, $N(9, 1)$. Translate ΔLMN using vector $\langle -2, 6 \rangle$.

Translation is (x, y)→(x-2, y+6)
L'(0, 8), M'(3, 9), N'(7, 7)

4.1 Translations

- Write a rule for the translation of ΔPQR to $\Delta P'Q'R'$.



$(x, y) \rightarrow (x + 2, y - 1)$

4.1 TRANSLATIONS

- Draw ΔRST with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, -2)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x + 1, y + 2)$. Graph the image using prime notation.

R’(3, 4), S’(6, 4), T’(4, 0)

4.1 TRANSLATIONS

- Rigid Motion
  - Transformation that preserves length and angle measure.
  - A congruence transformation

Translation Theorem
A translation is a rigid motion.

4.1 Translations

- Composition of Transformations
  - Two or more transformations combined into a single transformation

Composition Theorem

The composition of two (or more) rigid motions is a rigid motion.

4.1 Translations

- Graph $\overline{RS}$ with endpoints $R(-8, 5)$ and $S(-6, 8)$. Graph its image after the composition.
  Translation: $(x, y) \rightarrow (x - 1, y + 4)$
  Translation: $(x, y) \rightarrow (x + 4, y - 6)$

- 172 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 42, 43, 44, 48, 52

Translation 1: $(x, y) \rightarrow (x - 1, y + 4)$
R(−8, 5) → R′(−8 − 1, 5 + 4)= R′(−9, 9)
S(−6, 8) → S′(−6 − 1, 8 + 4) = S′(−7, 12)

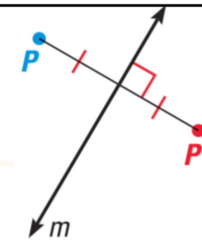
Translation 2: $(x, y) \rightarrow (x + 4, y - 6)$
R′(−9, 9) → R′(−9 + 4, 9 − 6) = R′(−5, 3)
S′(−7, 12) → S′(−7 + 4, 12 − 6) = S′(−3, 6)

After this lesson…

- I can reflect figures.
- I can perform compositions with reflections.
- I can identify line symmetry in polygons.

4.2 REFLECTIONS

4.2 REFLECTIONS



- Reflection
  - Transformation that uses a line like a mirror to reflect an image.
  - That line is called **Line of Reflection**
  - P and P' are the same distance from the line of reflection
  - The line connecting P and P' is perpendicular to the line of reflection

4.2 Reflections

- Graph a reflection of ΔABC where $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$ in the line x = 2.

New points are A(3, 3), B(-1, 2), C(2, 1)

4.2 Reflections

- Coordinate Rules for Reflections
  - Reflected in x-axis: (a, b) → $(a, -b)$
  - Reflected in y-axis: (a, b) → $(-a, b)$
  - Reflected in $y = x$: (a, b) → (b, a)
  - Reflected in $y = -x$: (a, b) → $(-b, -a)$

Reflection Theorem

A reflection is an rigid motion.

4.2 REFLECTIONS

- Graph ΔABC with vertices $A(1, 3)$, $B(4, 4)$, and $C(3, 1)$. Reflect ΔABC in the lines $y = -x$ and $y = x$.

y = -x: new points A(-3, -1), B(-4, -4), C(-1, -3)
y = x: new points A(3, 1), B(4, 4), C(1, 3)

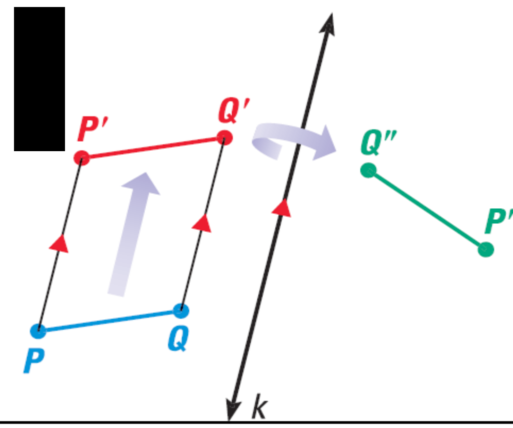
4.2 Reflections

- The vertices of ΔLMN are $L(-3, 3)$, $M(1, 2)$, and $N(-2, 1)$. Find the reflection of ΔLMN in the y-axis.

$(a, b) \rightarrow (-a, b)$
L(−3, 3) → L'(3, 3)
M(1, 2) → M'(−1, 2)
N(−2, 1) → N'(2, 1)

4.2 Reflections

- Composition of Transformations
  - Two or more transformations combined into a single transformation
- Glide Reflection
  - Translation followed by reflection over a line parallel to the translation



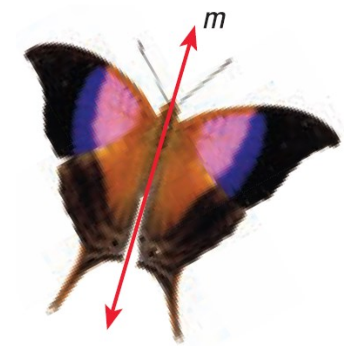
4.2 Reflections

- The vertices of ΔABC are $A(3, 2)$, $B(-1, 3)$, and $C(1, 1)$. Find the image of ΔABC after the glide reflection.
  Translation: $(x, y) \rightarrow (x, y - 4)$
  Reflection: Over y-axis

New points after translation: A’(3, -2), B’(-1, -1), C’(1, -3)
New points after reflection: A’’(-3, -2), B’’(1, -1), C’’(-1, -3)

4.2 Reflections

- Line symmetry
  - The figure can be mapped to itself by a reflection
  - The line of reflection is called **Line of Symmetry**



- Humans tend to think that symmetry is beautiful

4.2 Reflections

- How many lines of symmetry does the object appear to have?



- 180 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 45, 49, 51, 54, 55

Flower: 4 lines of symmetry
Sea Star: 5 lines of symmetry
Goat: 1 line of symmetry

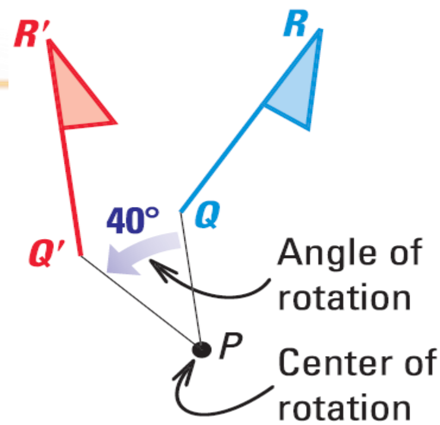
After this lesson…

- I can rotate figures.
- I can perform compositions with rotations.
- I can identify rotational symmetry in polygons.

4.3 ROTATIONS

4.3 Rotations

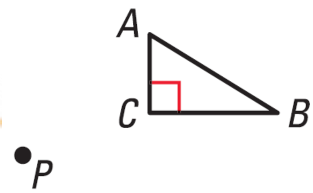
- Rotation
  - Figure is turned about a point called **center of rotation**
  - The amount of turning is **angle of rotation**



Rotation Theorem

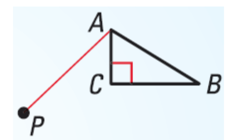
A rotation is a rigid motion.

4.3 Rotations

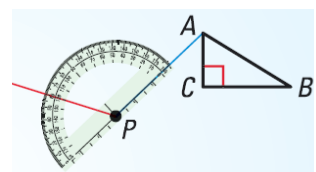


- Draw a $120°$ rotation of ΔABC about P.

1. Draw a segment from A to P.

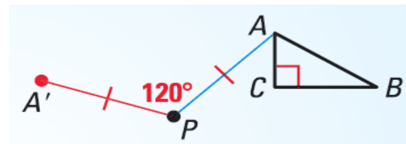


2. Draw a ray to form a $120°$ angle with $\overline{PA}$

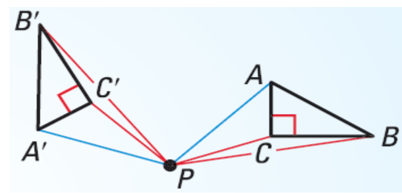


4.3 Rotations

3. Draw A' so that $PA' = PA$

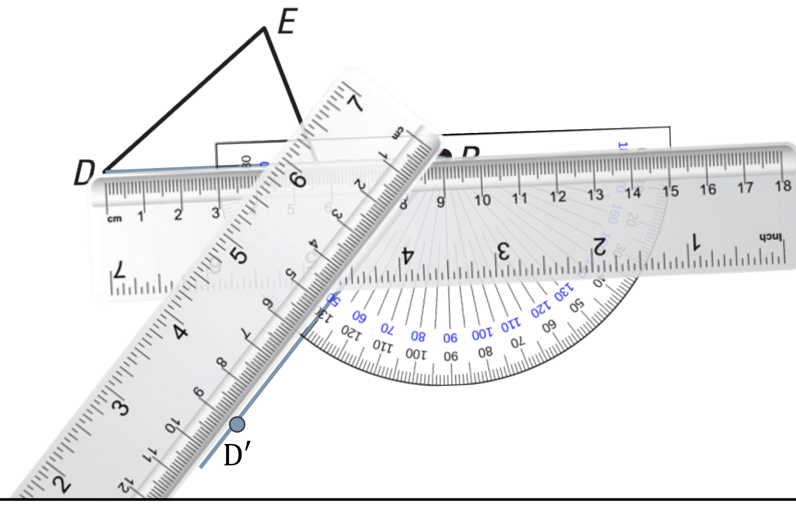


4. Repeat steps 1-3 for each vertex. Draw $\Delta A'B'C'$.



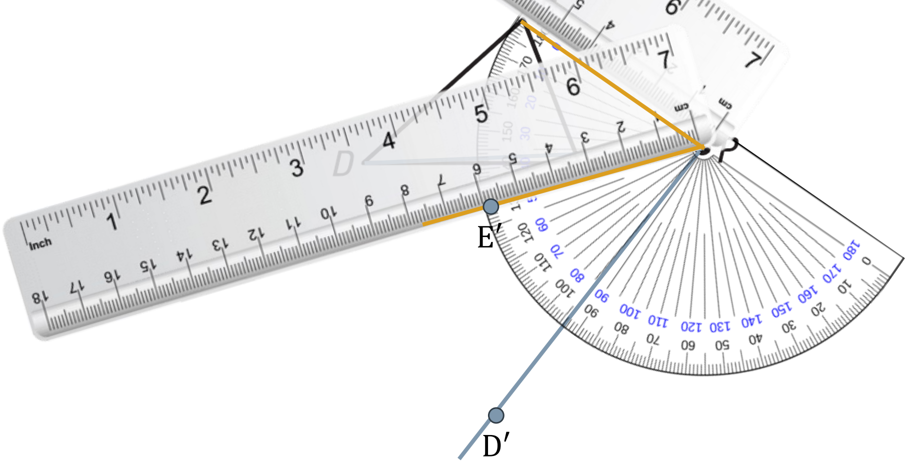
4.3 ROTATIONS

- Draw a 50° counterclockwise rotation of ΔDEF about P.



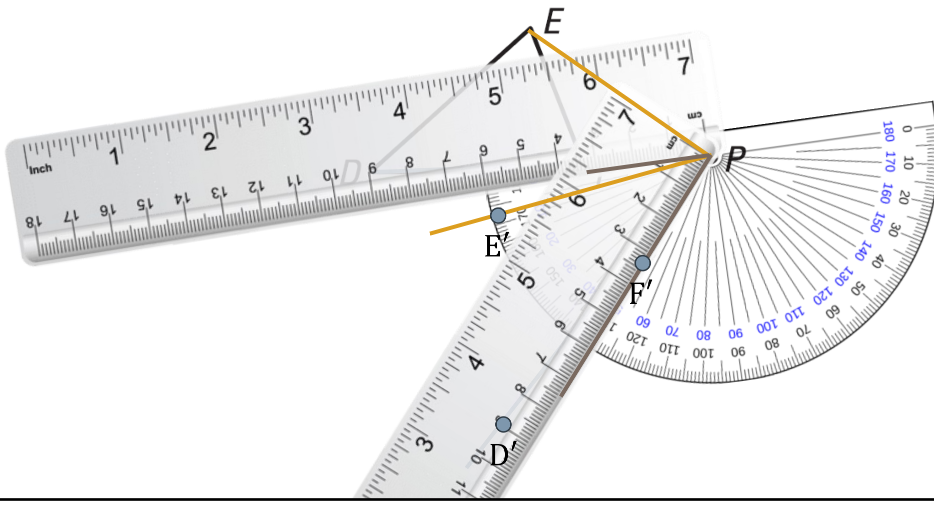
4.3 Rotations

- Draw a 50° counterclockwise rotation of ΔDEF about P.



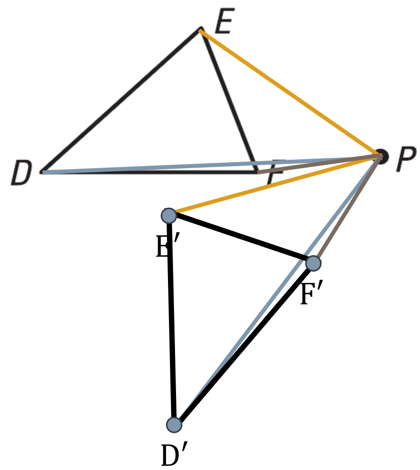
4.3 Rotations

- Draw a 50° counterclockwise rotation of ΔDEF about P.



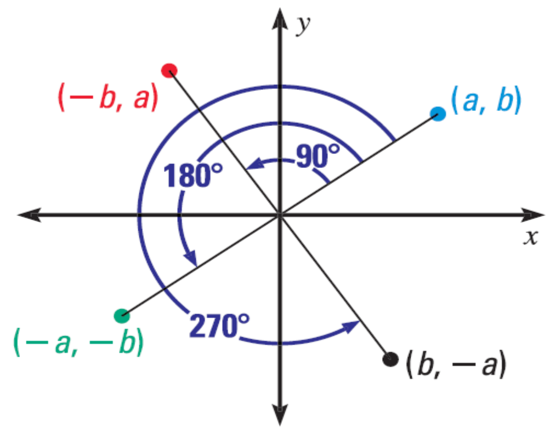
4.3 Rotations

- Draw a $50°$ counterclockwise rotation of ΔDEF about P.



4.3 Rotations

- Coordinate Rules for Counterclockwise Rotations about the Origin
- 90°: $(a, b) \rightarrow (-b, a)$
- 180°: $(a, b) \rightarrow (-a, -b)$
- 270°: $(a, b) \rightarrow (b, -a)$



4.3 Rotations

- If $E(-3, 2)$, $F(-3, 4)$, $G(1, 4)$, and $H(2, 2)$. Find the image for a $270°$ rotation about the origin.

(a, b) → (b, −a)
E(−3, 2) → E'(2, 3)
F(−3, 4) → F'(4, 3)
G(1, 4) → G'(4, −1)
H(2, 2) → H'(2, −2)

4.3 ROTATIONS

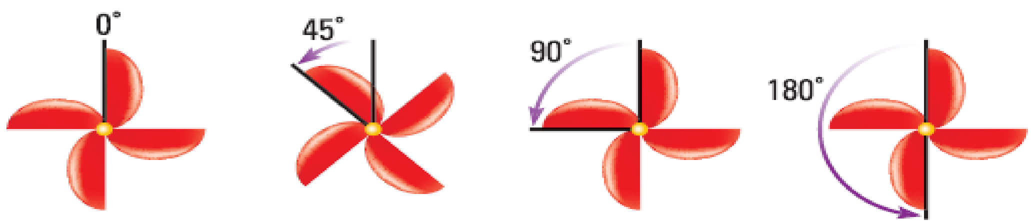
- Graph $\overline{RS}$ with endpoints $R(1, -3)$ and $S(2, -6)$ and its image after the composition.
  Rotation: 180° about the origin
  Reflection: in the y-axis

Rotation: (a, b) → (-a, -b)
R(1, −3) → R′(−1, 3)
S(2, −6) → S′(−2, 6)

Reflection: (a, b) → (-a, b)
R′(−1, 3) → R″(1, 3)
S′(−2, 6) → S″(2, 6)

4.3 Rotations

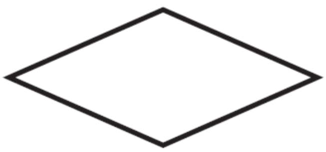
- Rotational Symmetry
  - The figure can be mapped to itself by a rotation of 180° or less about the center of the figure
  - The center of rotation is called the **Center of Symmetry**



Note: the 45° is not a symmetry

4.3 ROTATIONS

- Does the figure have rotational symmetry? What angles?



- 188 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 40, 42, 43, 46, 47

Rhombus: 180°
Octagon: 90°, 180°
Triangle: none

After this lesson…

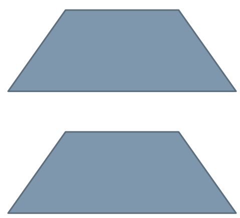
- I can identify congruent figures.
- I can describe congruence transformations.
- I can use congruence transformations to solve problems.

4.4 CONGRUENCE AND TRANSFORMATIONS

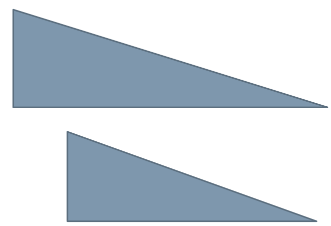
4.4 Congruence and Transformations

Congruent $\cong$

Exactly the same shape and size.



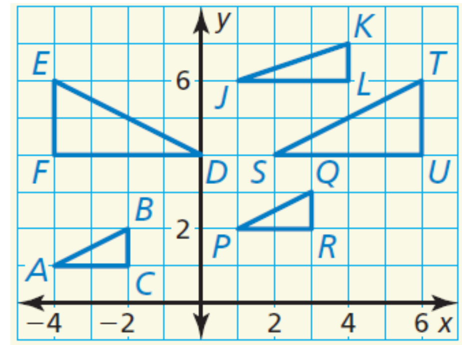
Congruent



Not Congruent

4.4 Congruence and Transformations

- Identify any congruent figures in the coordinate plane. Explain.



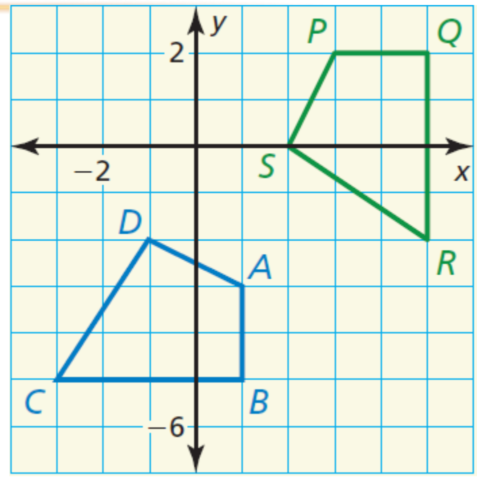
△*ABC* ≅ △*PQR*, △*DEF* ≅ △*STU*; △*PQR* is a translation 5 units right and 1 unit up of △*ABC*. △*STU* is a reflection of △*DEF* in the line $x = 1$.

4.4 Congruence and Transformations

- Congruence Transformation
  - Transformation with rigid motion
  - Preimage ≅ image

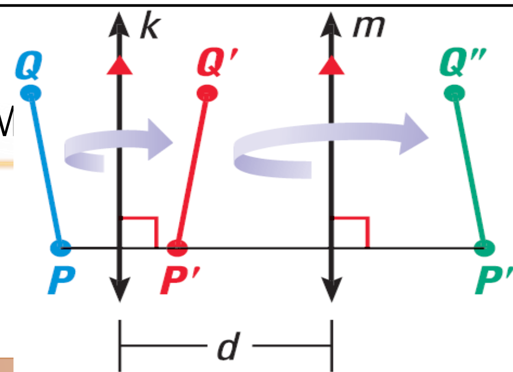
4.4 Congruence and Transformations

- Describe a congruence transformation that maps quadrilateral *ABCD* to quadrilateral *PQRS*.



90° rotation about the origin, followed by a translation 1 unit up

4.4 Congruence and Transform



Reflections in Parallel Lines Theorem

If lines *k* and *m* are parallel, then a reflection in line *k* followed by a reflection in line *m* is the same as a translation.

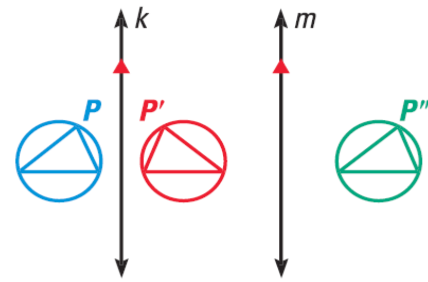
If P'' is the image of P, then

1. $\overline{PP''}$ is ⊥ to *k* and *m*, and
2. $PP'' = 2d$ where *d* is the distance between *k* and *m*

4.4 Congruence and Transformations

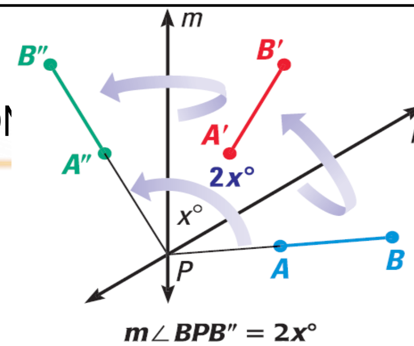
Use the figure below. The distance between line k and m is 1.6 cm.

1. The preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure to the green.
2. What is the distance from P and P''?



1. Translation in the x direction
2. 2(1.6 cm) = 3.2 cm (Reflections in Parallel Lines Thrm)

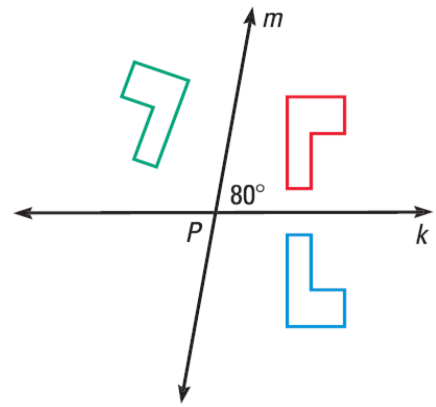
4.4 Congruence and Transformations



Reflections in Intersecting Lines Theorem
If lines *k* and *m* intersect at point P, then a reflection in line *k* followed by a reflection in line *m* is the same as a rotation about point P.
The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed *k* and *m*.

4.4 Congruence and Transformations

- In the diagram, the preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure to the green.



- 196 #2, 4, 6, 8, 10, 12, 14, 15, 16, 18, 20, 24, 26, 28, 35, 36, 42, 46, 49, 50

Counterclockwise rotation of 160° about point P

After this lesson…

- I can identify dilations.
- I can dilate figures.
- I can solve real-life problems involving scale factors and dilations.

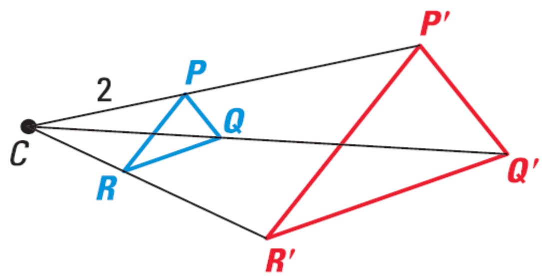
4.5 DILATIONS

4.5 Dilations

- Dilation
  - Enlarge or reduce
  - Image is similar to preimage
  - Scale factor is k
    - If $0 < k < 1$, then reduction
    - If $k > 1$, then enlargement

4.5 DILATIONS

- The image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$ and $k \neq 1$



- Scale factor is $\frac{\text{image}}{\text{preimage}}$

Scale

4.5 Dilations

- Coordinate Rule for Dilations
  - $(x, y) \rightarrow (kx, ky)$
  - Where k is the scale factor

4.5 Dilations

- Graph $\triangle PQR$ with vertices $P(4, 6)$, $Q(-4, 2)$, and $R(2, -6)$ and its image after a dilation with a scale factor of 0.5.

(x, y) → (kx, ky)
P(4, 6) → P'(0.5·4, 0.5·6) = P'(2, 3)
Q(−4, 2) → Q'(0.5·−4, 0.5·2) = Q'(−2, 1)
R(2, −6) → R'(0.5·2, 0.5·−6) = R'(1, −3)

4.5 Dilations

- Draw and label ΔRST, then construct a dilation of ΔRST with R as the center of dilation and a scale factor of 3.

1. Draw ΔRST, then draw rays $\overrightarrow{RS}$ and $\overrightarrow{RT}$
2. Using a ruler, measure RS. Multiply by the scale factor. Using the ruler mark this length RS' on $\overrightarrow{RS}$. Repeat for the other rays.
3. Draw $\Delta R'S'T'$

4.5 Dilations

- You are using a magnifying glass that shows the image of an object as three times the object's actual size. Determine the actual length of a spider when the image of the spider seen through the magnifying glass is 6.75 centimeters long.

- 204 #2, 4, 6, 8, 10, 14, 16, 18, 20, 22, 24, 26, 28, 34, 38, 50, 52, 55, 56, 59

$$k = 3 = \frac{image}{preimage}$$
$$3 = \frac{6.75}{preimage}$$
$$3 \cdot preimage = 6.75$$
$$preimage = 2.25\ cm$$

After this lesson…

- I can perform similarity transformations.
- I can describe similarity transformations.
- I can prove that figures are similar.

4.6 SIMILARITY AND TRANSFORMATIONS

4.6 Similarity and Transformations

- Similar figures
  - Same shape; different sizes

- Similarity Transformation
  - Dilation or
  - Composition of dilation and another transformation

4.6 Similarity and Transformations

- Graph $\triangle ABC$ with vertices $A(12, -6)$, $B(0, -3)$, and $C(3, -9)$ and its image after the similarity transformation.
  Reflection: in the y-axis
  Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

Reflection: (x, y) → (−x, y)
A(12, −6) → A'(−12, −6)
B(0, −3) →B'(0, −3)
C(3, −9) → C'(−3, −9)

Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

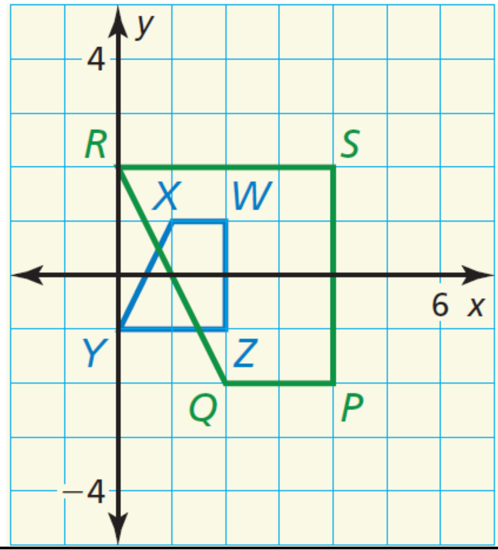
$$A'(-12, -6) \rightarrow A''\left(\frac{1}{3}(-12), \frac{1}{3}(-6)\right) = A''(-4, -2)$$

$$B'(0, -3) \rightarrow B''\left(\frac{1}{3}(0), \frac{1}{3}(-3)\right) = B''(0, -1)$$

$$C'(-3, -9) \rightarrow C''\left(\frac{1}{3}(-3), \frac{1}{3}(-9)\right) = C''(-1, -3)$$

4.6 SIMILARITY AND TRANSFORMATIONS

- Describe a similarity transformation that maps trapezoid *WXYZ* to trapezoid *PQRS*.



- 211 #2, 4, 6, 8, 10, ~~12,~~ **13**, 14, 16, 17, 19, 21, 22, 23, 24, 28

Sample answer: reflection in the *x*-axis, followed by a dilation with a scale factor of 2